Equivalent Static Loads Method for Non Linear Static Response Structural Optimization

Gyung-Jin Park

Professor, Department of Mechanical Engineering, Hanyang University, Ansan City, Korea

Summary:

Linear static response structural optimization using the finite element method for linear static analysis has been significantly developed. However, there is very little development of structural optimization where a non linear static analysis technique is required. To solve various structural optimization problems based on non linear analyses, the Equivalent Static Loads method for non linear static response Structural Optimization (ESLSO) has been proposed. ESLSO is a structural optimization method where equivalent static loads (ESLs) are utilized as external loads for linear static response optimization. The ESL is defined as the static load that generates the same displacement field by an analysis which is not linear static. An analysis that is not linear static is carried out to evaluate the displacement field. ESLs are evaluated from the displacement field, linear static response optimization is performed by using the ESLs, and the design is updated. This process proceeds in a cyclic manner. There are various ESLs methods according to the characteristics of the problems. In this paper, ESLSO using ESLs for displacement is introduced and various case studies are demonstrated.

Keywords:

Equivalent Static Loads Method for Non Linear Static Response Structural Optimization (ESLSO), Equivalent static loads, Structural optimization, Crashworthiness optimization, Metal forming, Forging

1 Introduction

Structural optimization has been significantly developed based on the development of computational analysis [1-3]. According to the characteristics of the design variables, structural optimization is classified into size, shape and topology optimizations. These techniques are excellent and have a wide variety of applications. Structural optimization is generally linear static response optimization because the involved finite element analysis (FEA) is linear static analysis [4]. However, computational analysis methods which are not linear static response analysis are frequently required in academics and industries because complex phenomena should be solved for the design and evaluation of engineering systems. In other words, the structural optimization methods which require non linear static analysis are necessary and these methods are named as non linear static response optimization. Examples of non linear static response optimization would be linear dynamic response optimization, structural optimization for multi-body dynamic systems, structural optimization for flexible multi-body dynamic systems, nonlinear static response optimization and nonlinear dynamic response optimization [5].

Some researchers have tried to solve non linear static response optimization problems by using conventional techniques which have been used in linear static response optimization [6-15]. When the conventional method is utilized, the cost is extremely high because sensitivity analysis is quite expensive. In many researches, approximation methods such as the response surface method (RSM) and the Kriging method, etc. [16-19] are utilized for non linear static response optimization. These methods only require function analysis using FEA and do not require any sensitivity analysis. However, these methods have two disadvantages. If we have many design variables, the computational cost is increased because a lot of function calculations are required. Since optimization is performed with approximated functions, the solution may not be accurate. Especially, when the quality of approximation is low, we may not obtain a good solution.

The Equivalent Static Loads method for non linear static response Structural Optimization (ESLSO), which has been proposed by Park, is a structural optimization method to solve non linear static response optimization problems [3, 5, 20]. The equivalent static loads (ESLs) should be calculated in the method. To calculate ESLs, non linear static analysis is performed and the displacement field is calculated from the results of this analysis. An ESL set is calculated by multiplying the linear stiffness matrix and the displacement field. Multiple ESL sets can be made according to the characteristics of the non linear static analysis and these are used as the external loads in linear static response optimization. A new design is found and the design variables are updated. Non linear static analysis is performed again with the updated design and the process proceeds in a cyclic manner. The optimization formulations are defined for some examples, the formulated problems are solved and the results are discussed. Non linear static analyses are performed using the commercial software LS-DYNA [21]. Calculation of the ESLs and linear response optimization are performed using commercial software systems [22, 23]. An interface program for LS-DYNA and the commercial optimization system is developed.

2 The Equivalent Static Loads Method for Non Linear Static Response Structural Optimization

ESLSO is used to optimize the non linear static response optimization problem such as linear dynamic response optimization, structural optimization for multi-body dynamic systems, structural optimization for flexible multi-body dynamic systems, nonlinear static response optimization and nonlinear dynamic response optimization.

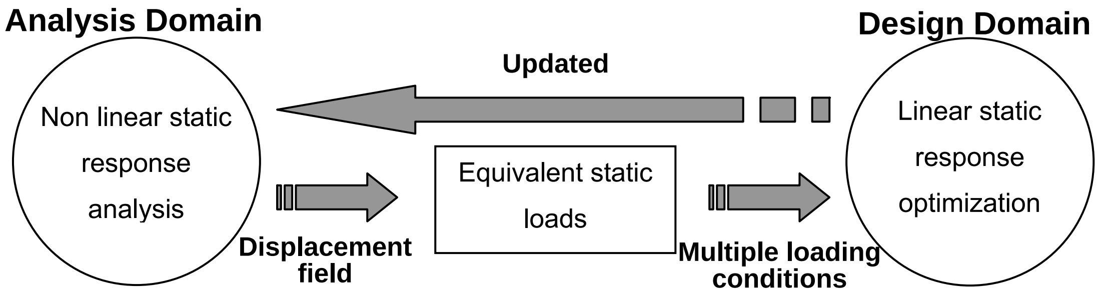


Fig. 1: Schematic process between the analysis domain and the design domain

The basic idea of ESLSO is presented in Fig. 1 [20]. The optimization process is divided into two domains: analysis domain and design domain. Non linear static analysis is performed in the analysis domain. The displacement field is evaluated and the equivalent load sets are derived from the displacement field. The equivalent load sets are transmitted to the design domain. In the design domain, linear static response optimization is performed by using the equivalent loads as external loads. The displacement field of the non linear static analysis is identical with that of the linear static response analysis of the first iteration in the design domain. The design variables are updated in the design domain and non linear static analysis is performed again with the updated design variables.

The process proceeds until the convergence criteria are satisfied. This process can be viewed as a cyclic process between a high fidelity model and a low fidelity model. The high fidelity model is solved in the analysis domain and the low fidelity model is considered in the design domain [3, 20].

According to the characteristics of the objective function or the constraint, ESLs are classified into the ESLs for the displacement [20, 24-31], the stress [32], the effective strain [33], the strain energy and the HIC [34]. In this chapter, nonlinear dynamic response optimization using ESLSO which uses ESLs for the displacement is briefly explained because this optimization is representative among the ESLSO methods.

2.1 Definition of the ESLs for the displacement

The ESLs are defined as the static loads which generate the same displacement fields as those under a dynamic load at an arbitrary time of dynamic analysis. In the finite element method, the governing equation of a structure in the time domain with nonlinearity is

$$\mathbf{M}(\mathbf{b},\mathbf{z}_N(t))\ddot{\mathbf{z}}_N(t)+\mathbf{C}(\mathbf{b},\mathbf{z}_N(t))\dot{\mathbf{z}}_N(t)+\mathbf{K}(\mathbf{b},\mathbf{z}_N(t))\mathbf{z}_N(t)=\mathbf{f}(t) \qquad (t=t_0,\, t_1,\, \cdots,\, t_l) \tag{1}$$

where $\mathbf{M}$ is the mass matrix which is the function of the design variable vector $\mathbf{b}$ and the nodal displacement vector $\mathbf{z}$. $\mathbf{C}$ is the damping matrix and $\mathbf{K}$ is the stiffness matrix which are also the function of $\mathbf{b}$ and $\mathbf{z}$. $\ddot{\mathbf{z}}$ is the acceleration vector and $\dot{\mathbf{z}}$ is the velocity vector. The constant l is the total number of the time steps for integration. $\mathbf{f}(t)$ is the external load vector and t is the time. The subscript N means nonlinear response analysis. $\mathbf{z}_N$ at all the time steps is obtained from Eq. (1).

The equivalent static load vector for displacements is defined as follows:

$$\mathbf{f}_{eq}^{\mathbf{z}}(s)=\mathbf{K}_L(\mathbf{b})\mathbf{z}_N(t) \qquad (s=1,\, 2,\, \cdots,\, l) \tag{2}$$

where L means linear static response analysis and the new notation s exactly matches with t. The reason the notation s is used is that Eq. (2) is not defined in the time domain. In other words, $t=t_j$ is equal to $s=j$ and the total number of s is l. Therefore, l equivalent static load vectors are obtained from Eq. (2). $\mathbf{f}_{eq}^{\mathbf{z}}(s)$ is the equivalent load vector for displacement at each time step. $\mathbf{K}_L$ is the linear stiffness matrix and $\mathbf{z}_N(s)$ is the nodal displacement vector from Eq. (1). $\mathbf{f}_{eq}^{\mathbf{z}}(s)$ is used in linear static analysis as follows:

$$\mathbf{K}_L(\mathbf{b})\mathbf{z}_L(s)=\mathbf{f}_{eq}^{\mathbf{z}}(s) \tag{3}$$

where the nodal displacement vector $\mathbf{z}_L(s)$ has the same values as the nonlinear nodal displacement $\mathbf{z}_N(t)$ in Eq. (1) at an arbitrary time. Therefore, if the equivalent static load $\mathbf{f}_{eq}^{\mathbf{z}}(s)$ is used as an external load in linear static response optimization, the same displacements as those of the nonlinear dynamic response analysis can be considered in linear static response optimization. ESLs are used as multiple loading conditions in linear static response optimization. It is noted that the multiple loading conditions can be easily handled in linear static response optimization.

2.2 ESLSO process using the ESLs for displacement

As mentioned earlier, the entire optimization process iterates between the two domains in Fig. 1 until the convergence criterion are satisfied. Fig. 2 shows the optimization process using the equivalent static loads for the displacements and the steps of the process are as follows:

Step 1 Set initial design variables and parameters (cycle number: $k=0$, design variables: $\mathbf{b}^{(k)}=\mathbf{b}^{(0)}$ convergence parameter: a small number ε).

Step 2 Perform non linear static analysis with $\mathbf{b}^{(k)}$.

Step 3 Calculate the equivalent static load sets.

Step 4 Solve the linear static response optimization problem with the equivalent load sets.

Step 5 When $k=0$, go to Step 6. When $k>0$, if $\left\|\mathbf{b}^{(k)}-\mathbf{b}^{(k-1)}\right\|\le\varepsilon$, then terminate the process. Otherwise, go to Step 6.

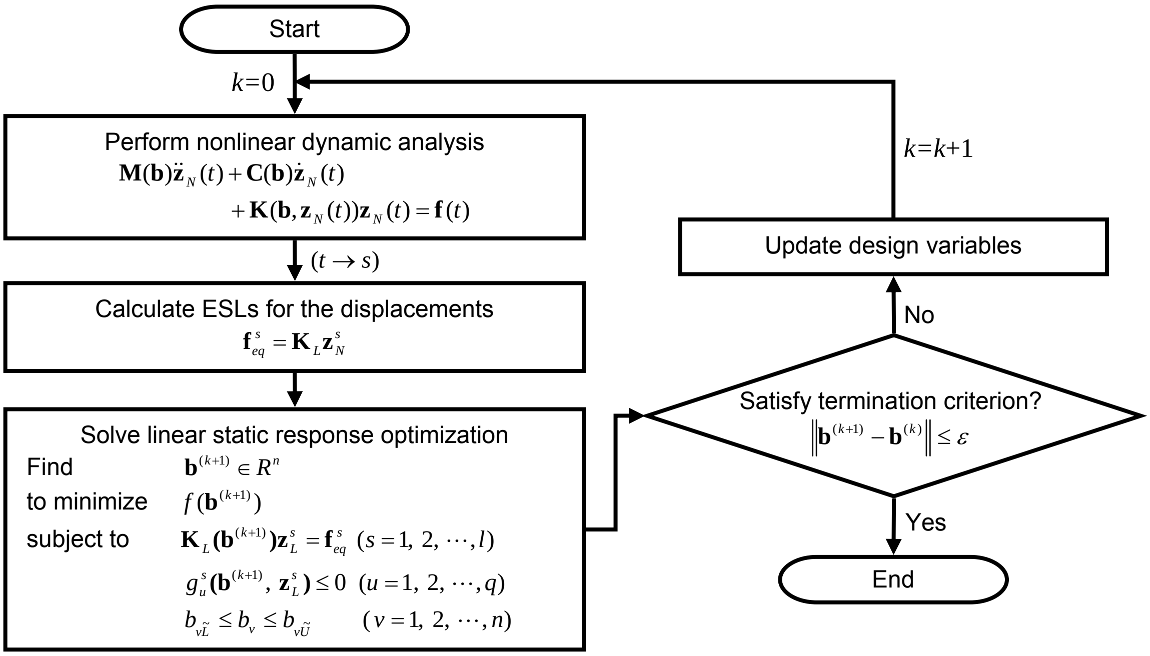


Fig. 2: ESLSO process using the ESLs for the displacements

Step 6 Update the design results, set $k = k + 1$ and go to Step 2.

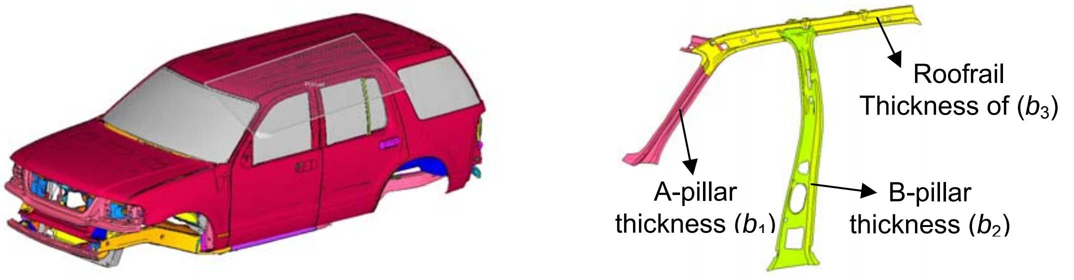
3 Applications

Six examples are solved using ESLSO. They are optimization of automobile structures and metal forming. These problems have three nonlinearities, which are geometric nonlinearity, material nonlinearity and boundary nonlinearity. And dynamic loads which depend on time are considered in the examples. Therefore, analyses are based on nonlinear dynamic analysis and the optimization process is nonlinear dynamic response optimization.

3.1 Optimization of an automobile roof

An automobile roof crush problem is solved using the ESLs for the displacements. The full FE model for the roof crush analysis is illustrated in Fig. 3(a). Three design variables are selected as illustrated in Fig. 3(b). The weight is minimized while a constraint is imposed on the Federal Motor Vehicle Safety Standard (FMVSS) No. 216 for roof crush resistance.

The results are shown in Table 1. It is noted that the problem is also solved by the RSM since it has a small number of design variables. The number of nonlinear dynamic analyses is considerably reduced compared to that of the RSM. A problem with nine design variables is solved as well. LSDYNA is used for nonlinear dynamic response analysis and NASTRAN is used for linear static response structural optimization [29].



(a) FE model (b) Important components for an automobile roof crush

Fig. 3: Automobile roof

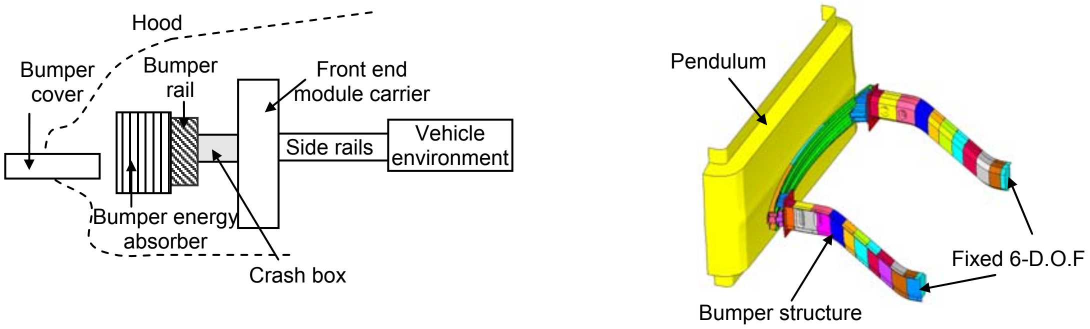
Table 1: Comparison of the optimization results for the roof crush problem

	Optimum value [mm]			Mass [kg]	Number of cycles	Constraint violation	Number of nonlinear analyses
	A-pillar	B-pillar	Roof-rail				
RSM	1.39	0.6	0.6	3.346	4	-11.2%	33
ESLSO	0.86	0.96	0.6	3.329	5	0.7%	5

3.2 Optimization of an automobile frontal structure

An automobile frontal structure is optimized using the ESLs for the displacements when a pendulum hits the bumper. A schematic view of the automobile structure is depicted in Fig. 4(a) and the FE model is illustrated in Fig. 4(b). It is a crashworthiness problem. It is noted that the impact occurs in an extremely short time and velocity and acceleration are involved. The problem has 28 design variables, the objective function is the mass of the structure and constraints are imposed on displacements, velocities and accelerations on some nodes.

The optimization process converges at the 6th cycle. The mass is decreased from 16.16kg to 15.66kg. But the constraint violation is increased from 0.26% to 2.79% due to the velocity constraint because the nonlinearity of velocity is more extreme than the other constraints. LSDYNA is used for nonlinear dynamic response analysis and GENESIS is used for linear static response structural optimization [30].



(a) Schematic view of the frontal structure (b) FE model of the frontal structure

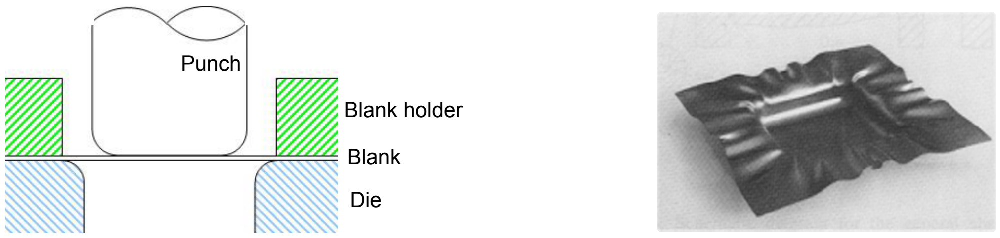
Fig. 4: Automobile frontal structure

3.3 Optimization of a straight square cup

A straight square cup is optimized using the ESLs for the displacements. A schematic view of the tooling for straight square cup forming is depicted in Fig. 5(a). When the blank holding force is weak, wrinkling occurs at the flange as illustrated in Fig. 5(a). The optimization model is a quarter of a square cup and the design variables are the scale factors of the perturbation vectors. The blank shape is controlled by these vectors. The objective function is wrinkling reduction at the flange of the final shape and the constraints are differences between the final shape of the deformed edge and the final desired shape.

The results are shown in Table 2. The final shape of the deformed edge is quite close to the final desired shape and the height at the flange is even. In other words, the constraints are satisfied and wrinkling is reduced. Therefore, the post processes for the wrinkling reduction and trimming are not needed after the forming process.

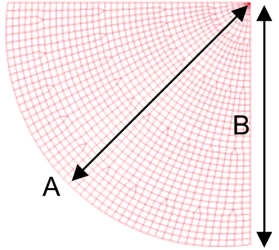
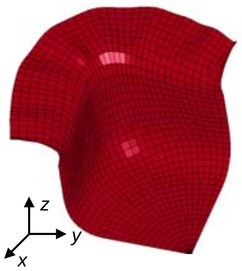
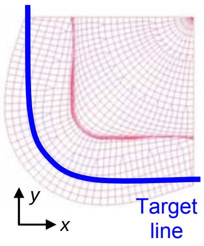
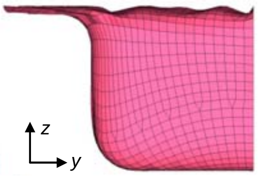
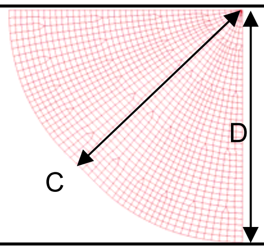
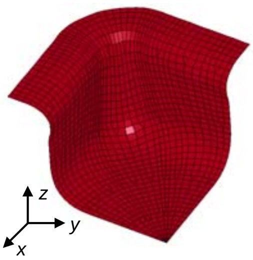
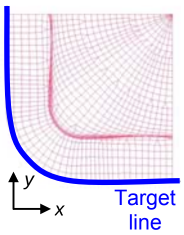
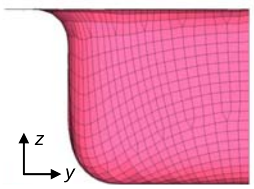
LSDYNA is used for nonlinear dynamic response analysis and NASTRAN is used for linear static response structural optimization. The sheet metal is a planer anisotropic material under plane stress conditions and the utilized yield criterion is the Barlat's yield criterion. The used material model in LS-DYNA is MAT_3-PARAMETER_BARLAT (MAT_036) which is used for modeling sheets with anisotropic materials under plane stress conditions [31].



(a) The tooling for the straight square cup (b) Deformed shape of a wrinkled straight square cup

Fig. 5: Straight square cup

Table 2: Shape comparison of the tapered square cup between the initial and the optimum model

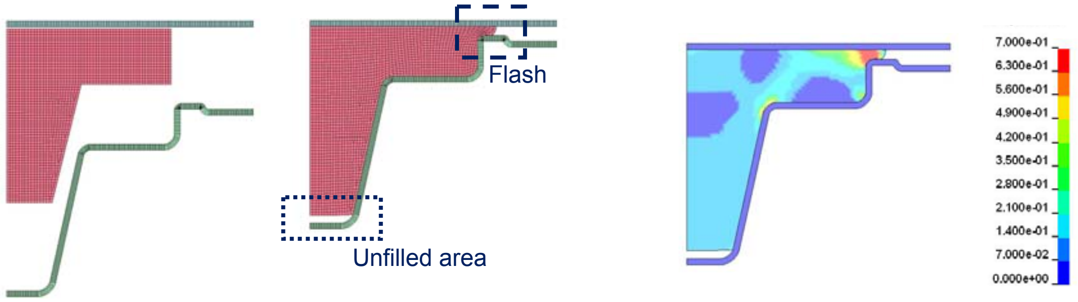
	Initial blank shape	Shape of blank after the sheet metal forming		
Initial model	 A: 90.0 mm, B: 85.0 mm			
Optimum model	 A: 80.4 mm, B: 83.7 mm			

3.4 Optimization of a preform of a T-shape forging

A preform of the T-shape forging is optimized using the ESLs for the effective strains. Fig. 6(a) presents the die and the shape of the initial preform and the model is half of the cross section of the T-shape forging. After forging, the unfilled area and flash occurs as illustrated in Fig. 6(b). And distribution of the effective strain is not even. Optimization of the preform shape is necessary for the even distribution of the effective strain and reduction of the unfilled area and flash.

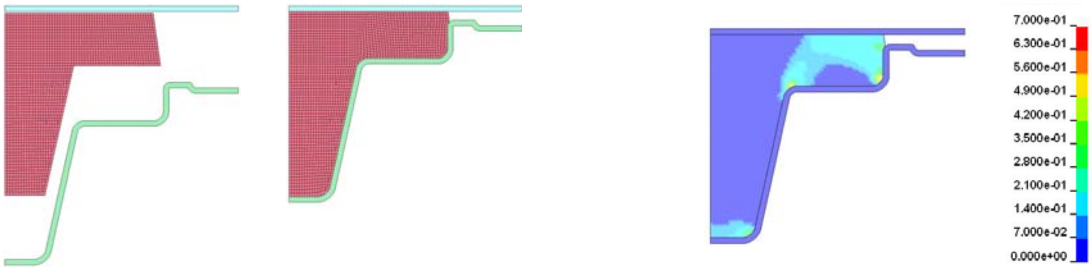
The optimization model is half of a T-shape forging and the design variables are the scale factors of the perturbation vectors. The preform is controlled by these vectors.The objective function is that the final forging has even distribution of the effective strain. The constraints are the reduction of the unfilled area and flash, and they are imposed by the desired shape.

Figure 7 shows the optimum model of the preform of the T-shape forging. The effective strain contour of the initial model and the optimum model. The maximum value of the effective strain of the initial model is 1.135 and the value of the optimum model is 0.607. And the unfilled area and flash do not occur. Therefore, the optimum model of the preform satisfies the desired final forging shape [33].



(a) Before forging (b) After forging (c) Effective strain contour of the initial model

Fig. 6: The initial model of the T-shape forging



(a) Before forging (b) After forging (c) Effective strain contour of the optimum model

Fig. 7: The optimum model of the T-shape forging

3.5 Optimization of a crash box of the vehicle

A crash box is optimized using the ESLs for the strain energy. The crash box is usually installed between the bumper and the side rail to absorb the impact energy in a RCAR test condition. Figure 8 illustrates the finite element model of the crash box with the simplified RCAR test condition. The crash box has an octagonal sectional shape constructed by three plates. The role of the crash box is absorbing the impact energy as much as possible, and it leads to preventing the interior parts of the vehicle from being damaged. The design variables are the thicknesses of the crash box. The objective function is the strain energy at the specific time. The constraints are imposed on displacement to prevent the collapse of the side rail.

The objective function is increased about 5.9% compared to the initial model while the constraints are satisfied within 10 cycles. In verifying the optimum solution of ESLSO, the results of ESLSO are compared with those of RSM. Table 3 shows the comparison of the optimized results of ESLSO and SRSM. The optimum strain energy values are similar and the constraint is active at the optimum of both methods. It is verified that the proposed ESLSO can find a reasonable local optimum with a small number of crash analyses [34].

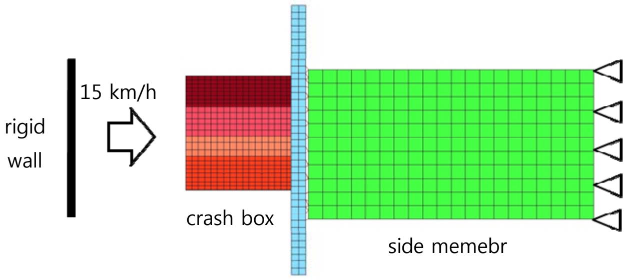


Fig. 8: Finite element model for designing the crash box

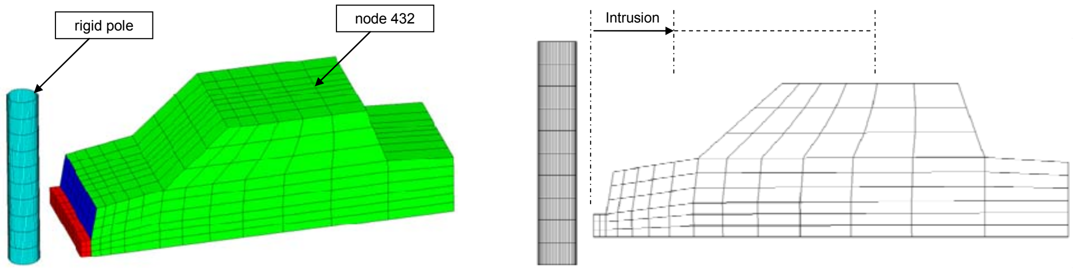
Table 3: Comparison of the optimization results for the crash box problem

	Optimum value [mm]			Strain energy [x10^6kJ]	Number of cycles	Constraint violation	Number of nonlinear analyses
	b_1	b_2	b_3				
RSM	1.12	1.21	1.24	2.828	8	0.40%	57
ESLSO	1.08	1.30	1.18	2.827	11	-0.28%	11

3.6 Optimization of a simplified vehicle structure

The last example is crashworthiness design optimization of the frontal structure of a simplified vehicle. It is performed to reduce head injury. Figure 9(a) shows the finite element model, which includes the simplified vehicle model and the rigid pole. The simplified vehicle model is optimized to minimize the HIC value of node 432, as shown in Fig. 9(a). Two design variables are selected, one is the thicknesses of the hood, front and underside and another is the thickness of the bumper. The intrusion condition is considered as the constraint as illustrated in Fig. 9(b).

The HIC value of the optimum model increases while satisfying the constraint condition. To verify the optimum solution of ESLSO, the results of ESLSO are compared with the result of SRSM. Table 4 shows the comparison of the optimized results of the ESLSO and the SRSM. The optimum objective function values are similar and the constraint is active at the optimum of both methods [34].



(a) FE model of the simplified vehicle model (b) Measurement of an intrusion

Fig. 9: Simplified vehicle model

Table 4: Comparison of the optimization results for the simplified vehicle problem

	Optimum value [mm]		HIC	Number of cycles	Constraint violation	Number of nonlinear analyses
	b_1	b_2				
RSM	1.84	5.00	167.0	4	0.02%	31
ESLSO	1.87	4.16	162.9	14	-0.06%	14

4 Conclusions

It is well known that optimization using non linear static analysis is extremely difficult and costly. ESLSO was proposed for non linear static response structural optimization. The response field is evaluated from non linear static analysis and ESLs are made. The ESLs are used as external loads for linear static response optimization and the design is updated. The updated design is utilized for a non linear static analysis and the process proceeds in a cyclic manner.

The examples are solved to verify the proposed method and the results are discussed. ESLs which are generated by characteristics of each problem are utilized to solve the examples. Also, the advantage of ESLSO is verified from the comparisons of the optimization results between ESLSO and RSM. The number of nonlinear dynamic analyses is considerably reduced compared to that of the RSM.

A representative problem of ESLSO is that the design variables, the objective function and constraints should be defined so as to use them in linear static response. Therefore, we cannot use design variables or functions which cannot be defined in linear static response optimization. However, some approximation methods such as the response surface method can use such design variables. This aspect should be improved in order to be more practical.

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