Determination of the Behaviour of Thermoplastics at High Strain-Rates Using the Invariant Theory

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Abstract:

For the finite element simulations of polymeric materials mathematical models are needed which cover all the phenomena of the material.

In the first part of this paper a new model based on the invariant theory is introduced. With this model it is possible to describe accurately the mechanical behaviour of thermoplastic materials. The second part shows methods for the data processing of the measured data. These data are used for the determination of the model's parameter. A new filter method is introduced which can reduce the superposed oscillations from the measured data.

Keywords:

high speed testing, strain rate, material model, invariant theory, DFT, filter methods

1   Introduction

Finite element simulation, close to reality, of parts made from thermoplastics a deeper knowledge about the material behaviour at different loads and environmental influences are necessary. Also there should be mathematical models to describe these influences according to the simulation problem.

The material models developed in the past are mostly based on the extensions of models originally made for other material classes like metals and the calculation of the static behaviour [1]. But, these models did not consider several phenomena of the polymeric materials [2]. For example models based on the Von-Mises plasticity do not differ on the behaviour of tension, compression or shear (Fig. 1.1). Furthermore, compressibility is neglected. There are other mathematical approaches like the DSGZ model [3] that is capable to calculate the behaviour of the two uniaxial load cases, tension and compression, in dependence of strain, strain rate and temperature. Again combinations of load cases are neglected. With the latest approach, the SAMP-1 model [4], the behaviour of thermoplastics can be calculated very close to reality at all multiaxial stress states.
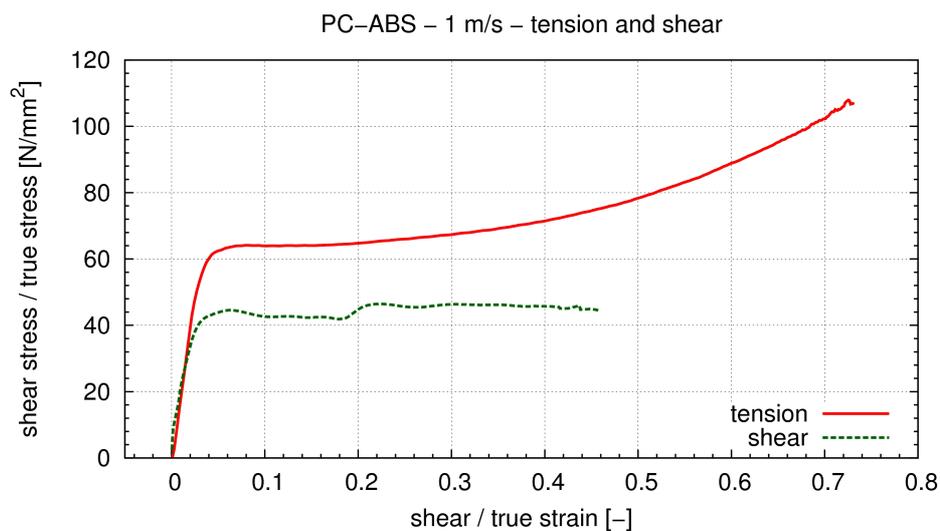
Fig. 1.1 Stress-strain curves for two different stress states of PC-ABS

The aim of the new model that is based on the invariant theory is to describe the multiaxial behaviour of thermoplastics including all specific phenomena in a parametric way. Therefore a one-dimensional description of the stress-strain curve and a potential law based on the invariant theory are combined by an associated yield law. For the description of the uniaxial stress-strain relation in dependence of strain, strain rate and temperature a modified model of *G'Sell-Jonas* is used [5,6]. The potential law for the multiaxial extension will be of third order according to *Kolupaev* [7].

This paper contains two parts. One is the description of a new material model for the calculation of the high strain rate behaviour of thermoplastic materials. The other part is the determination of the material properties. The attention here is on the signal processing to remove the oscillations in the force signal.

2   Material modelling

2.1   Associated flow rule

For the mathematical description of the multiaxial stress-strain behaviour at high strain rates the total strain is divided into an elastic and a plastic part (2.1) [8]. The elastic deformation is calculated by the *Hook's* law using the elasticity modulus and the *Poisson's* ration which is dependent on the occurring strain rate.

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^{el} + \boldsymbol{\epsilon}^{pl} \tag{2.1}$$

$$\boldsymbol{\epsilon}^{pl} = \lambda \cdot \frac{\partial \boldsymbol{\Phi}}{\partial \boldsymbol{\sigma}} \tag{2.2}$$

$$\epsilon_{11}^{pl} = \lambda \cdot \frac{\partial \boldsymbol{\Phi}}{\partial \sigma_{11}} \tag{2.3}$$

The plastic deformation is calculated by an associated flow rule (2.2). For this flow rule a potential law Φ exists which connects the strain and stress state in the material mathematically. The parameter λ can be defined through the one-dimensional approach and the flow rule in one direction (2.3).

2.2 Potential law

The potential law for an isotropic material can be expressed as a function of three invariants (2.4, 2.5) [8]. The power n has to be adapted for the special needs and accuracy of the material description.

$$I_1 = \sigma_{kk}, \quad I_2' = \frac{1}{2} s_{ij} s_{ji}, \quad I_3' = \frac{1}{3} s_{ij} s_{jk} s_{ki} \tag{2.4}$$

$$\Phi_n\left(I_1, I_2', I_3'\right) = \sigma_V^n \tag{2.5}$$

For the calculation of the material behaviour at high strain rates, a potential law of the third order (2.6) can be used.

$$\Phi_3 = \frac{1}{k^2} \cdot \left[\sigma_V \cdot (3 I_2' + a_1 \sigma_V I_1 + a_2 I_1^2) + a_3 I_1^3 + c_3 I_3' + d_3 I_1 I_2' \right] \tag{2.6}$$

The parameters $a1$ to $d3$ are determined by several material tests. The tests require the multiaxial analysis of the material properties at different stress states and exact determination of the occurring strains in the specimen.

2.3 One-dimensional approach

For the determination of the parameter λ (2.2) a one-dimensional model is needed. The origin of the used model is the approach of *G'Sell-Jonas* (2.7) [5,6]. With this model it is possible to describe the stress-strain behaviour of thermoplastic materials in dependence of strain and strain rate. Also the hardening of the material after yield stress can be described. There is also an extension to consider the temperature influences.

$$\sigma = K \cdot \left(1 - e^{-\left(\frac{\epsilon}{\epsilon_v}\right)}\right) \cdot e^{h \cdot \epsilon^2} \cdot \dot{\epsilon}^m \tag{2.7}$$

$$\sigma_1 = K \cdot \left(1 - e^{-\left(\frac{\epsilon}{\epsilon_v}\right)}\right), \quad \sigma_2 = e^{h \cdot \epsilon^2}, \quad \sigma_3 = \dot{\epsilon}^m \tag{2.8},$$

The model can be divided into three parts (2.8). The first part describes the material behaviour until yielding occurs. The second part is used to apply the strain hardening. The third one describes the strain rate dependency. Comparing to other models the *G'Sell-Jonas* approach shows a good result reproducing the stress-strain behaviour of the material in the whole strain area (Fig. 2.1). But there is still a gap near to the yield stress between the measured and the fitted data.
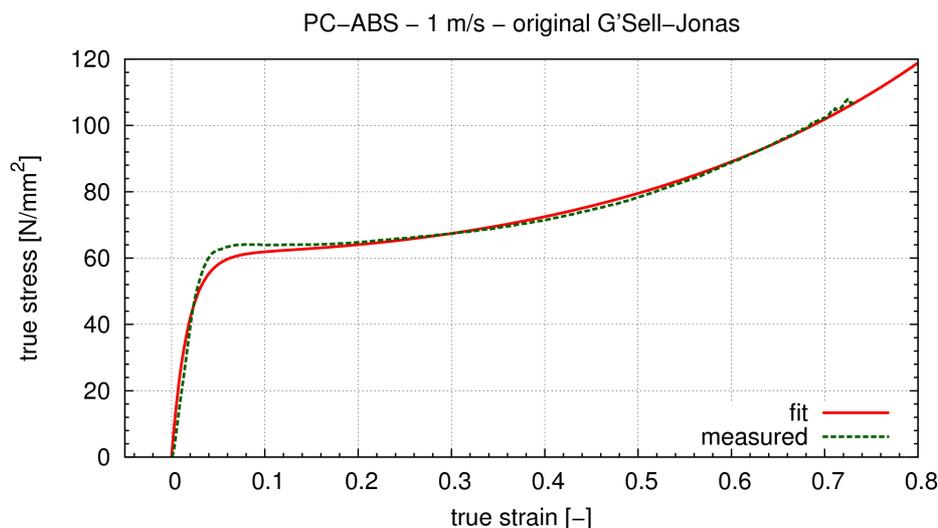
Fig. 2.1 Original model of G'Sell-Jonas describing PC-ABS at a deformation speed of 1 m/s

The first extension to reduce the deviation between measured and fitted data is to introduce two more parameters. The parameters t and u (2.2) expand the first and the second part to a variable power. These modifications make the equation more fittable in the first and second part.

$$\sigma = K \cdot \left(1 - e^{-\left(\frac{\epsilon}{\epsilon_v}\right)^t}\right) \cdot e^{h \cdot \epsilon^u} \cdot \dot{\epsilon}^m \tag{2.9}$$

If there is a yield-drop $(\sigma_y - \sigma_D)$ (Fig. 2.2) [9] the extended model is not fittable enough to reproduce this phenomena. Therefore the model (2.9) has to be extended (2.10). The parameters of this extension can be determined analytically using the equation 2.12.

$$\sigma_4 = \left(1 - p \cdot e^{-q \cdot (\epsilon - d)^2}\right) \tag{2.10}$$

$$\sigma = K \cdot \left(1 - e^{-\left(\frac{\epsilon}{\epsilon_v}\right)^t}\right) \cdot e^{h \cdot \epsilon^u} \cdot \left(1 - p \cdot e^{-q \cdot (\epsilon - \epsilon_D)^2}\right) \cdot \dot{\epsilon}^m \tag{2.11}$$

$$p = \frac{(\sigma_y - \sigma_D)}{\sigma_y} , \quad q = \frac{\ln(z/p)}{(\epsilon_y - d)^2} , \quad z \approx 0{,}005 \tag{2.12}$$
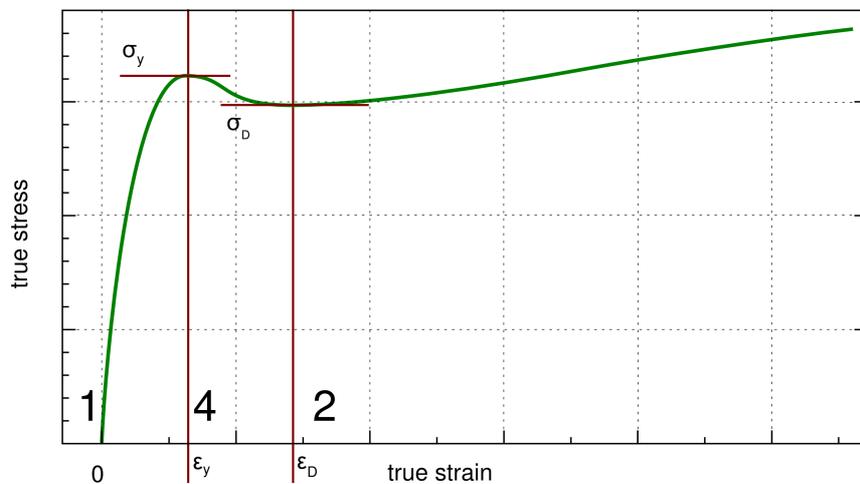
Fig. 2.2 Parts of the modified model of G'Sell-Jonas

Comparing the fitted data to the measured data (Fig. 2.3) one can conclude that the model is capable to reproduce the data for the one-dimensional calculation of the stress-strain behaviour of the material. Whether all extensions and parameters are used or not depends on the needed accuracy of the fit.
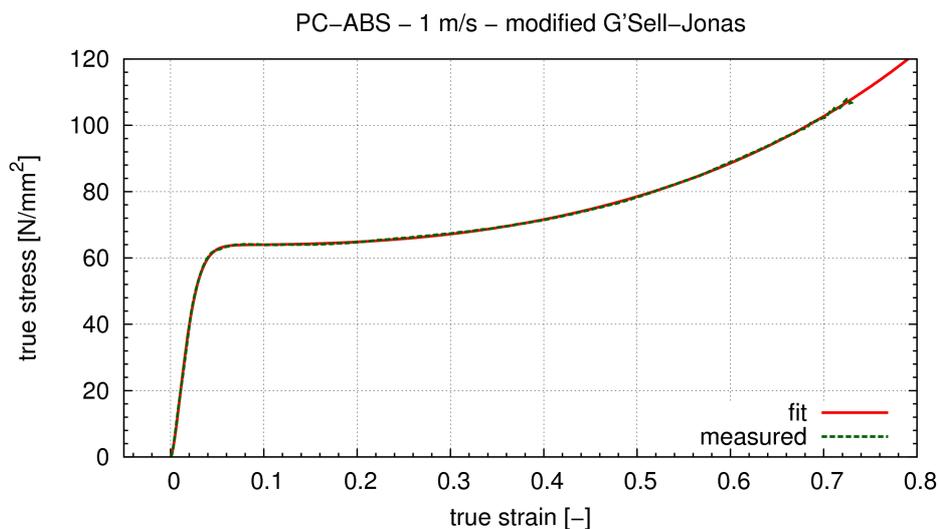
Fig. 2.3 Modified model of G'Sell-Jonas describing PC-ABS at a deformation speed of 1 m/s

3   Material characterisation

3.1   Experimental procedure for High-Speed tests

For the characterisation of the material the parameters of the material model have to be determined by material tests. Because of the complexity of the model, tests at different stress states have to be performed. Standard stress states are tension, shear and compression. Also biaxial and internal or external pressure stress states can be conducted.

Commonly for the material characterisation at high strain rates only tension and shear tests are performed. Therefore two specimen geometries were developed (Fig. 3.1). Both are made out of the same injection moulded bar geometry. If necessary this bar can be produced with a defined fibre orientation [10].
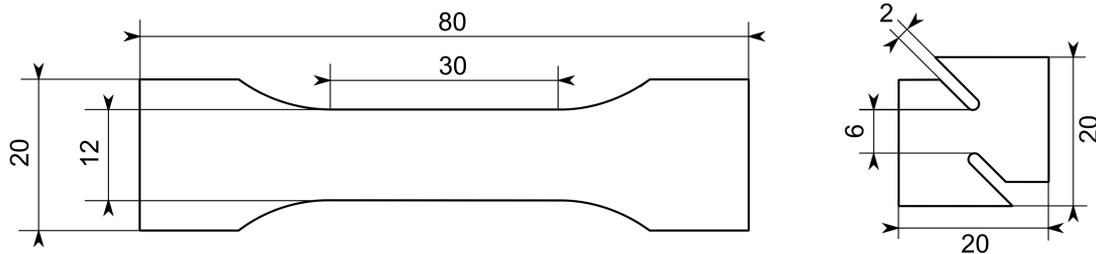
Fig. 3.1 Specimen geometries for tension and shear tests

The material tests to determine the parameters are performed on a servo-hydraulic high speed testing machine (Zwick-Amsler HTM5020) up to a maximum speed of 20 m/s. Strain measurement is realised by grey-scale correlation. To assure an accurate determination of the strain with the grey-scale correlation method the width of the specimen in the parallel area is 12 mm. The force is measured by a piezoelectric load cell at the static side of the specimen clamps.

3.2   Filter methods for data analysis

3.2.1   Common filter methods

As typical for high dynamical systems with less damping the measured force signal shows oscillations with high amplitude values (Fig. 3.3). This oscillations result from the natural frequencies of the fixed side of the clamping system. In the used system the main frequency is around 7 kHz shown in the frequency spectrum in Fig. 3.2. To get the real force in the specimen, the overlaying natural frequencies have to be removed. The reduction of these frequencies has to be done carefully not to distort the force signal.
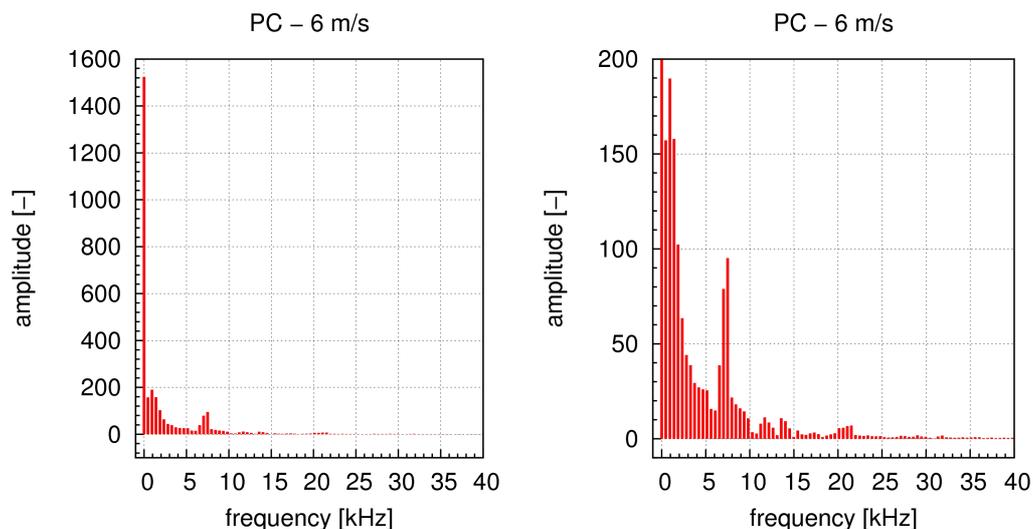
Fig. 3.2 Frequency spectrum of the force signal calculated by DFT:
whole spectrum (l), zoomed (r)

Common filters used to reduce such oscillations are low-pass or band-stop filters. Also Butterworth low-pass filters of several orders can be used. The results of a low-pass and a band-stop filter applied to the force signal are shown in Fig. 3.3. The low-pass filter in this example had a limit frequency of 6 kHz and the band-stop filter had a stopband between 5 kHz and 8.5 kHz. In both cases the filtered signal is better than the original one. But there are still artefacts of the oscillation left. In the beginning and at the end of the signal overshootings appear.

One can conclude that the filter methods can be used to reduce the oscillations but not to remove the oscillations. Therefore a more sophisticated method is needed to remove the superposed signal.
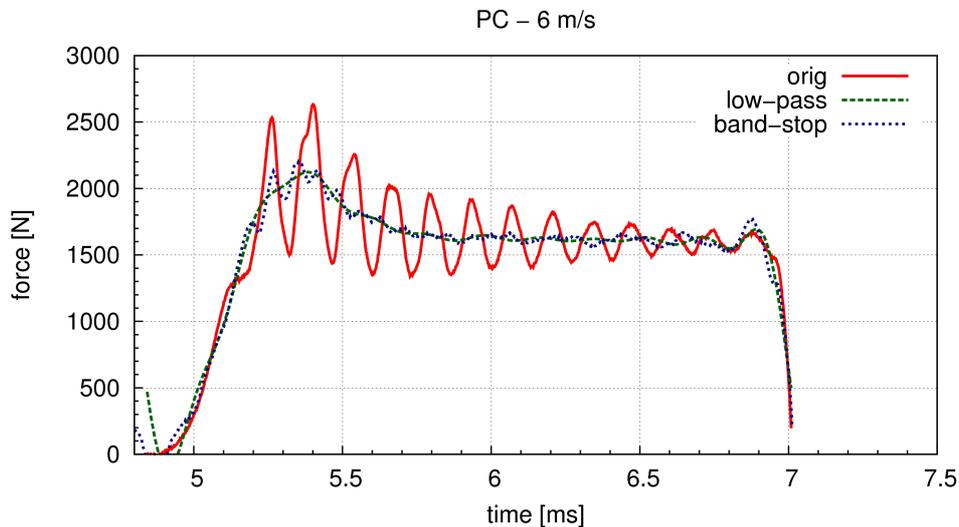
Fig. 3.3 Origin and filtered force signals for PC-ABS tested at 6 m/s
using several common filter methods in the frequency spectrum

3.2.2   Frequency filter approach

Filter methods shown in Fig. 3.3 produce a more or less rough decrease or steps in the frequency spectrum. These steps produce artefacts in the filtered force signal. Based on an ideal stress-strain curve (Fig. 3.5) the frequency spectrum can be determined. There should be no peaks or gaps in the amplitude slope. To prevent artefacts in the filtered signal the decrease in the frequency spectrum should be smooth like in Fig. 3.5.
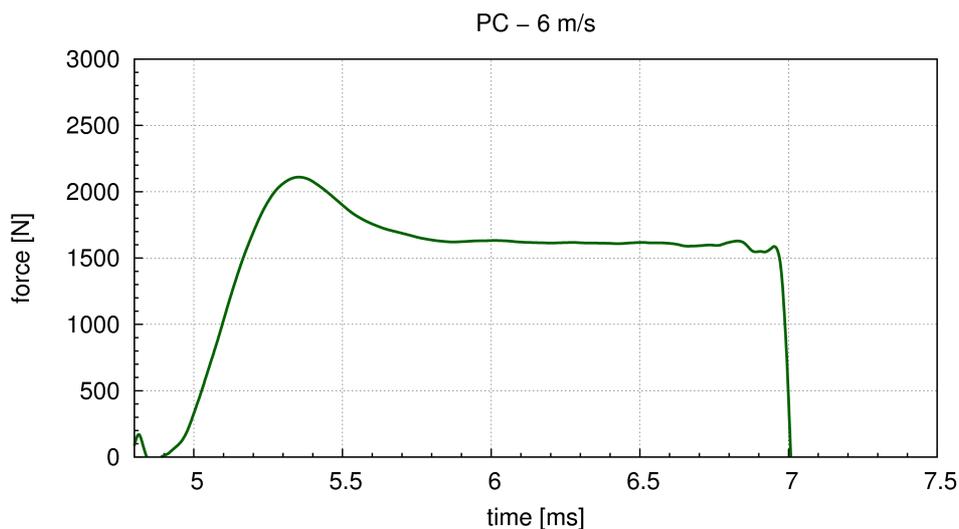
Fig. 3.4 Ideal force-time curve

To get such a smooth frequency spectrum from the real spectrum of Fig. 3.2 a smooth method is needed. At first the frequency spectrum has to be determined by discrete fourier transformation (DFT). Starting at this spectrum two different methods are possible: 2-point line and linear fit (Fig. 3.6).
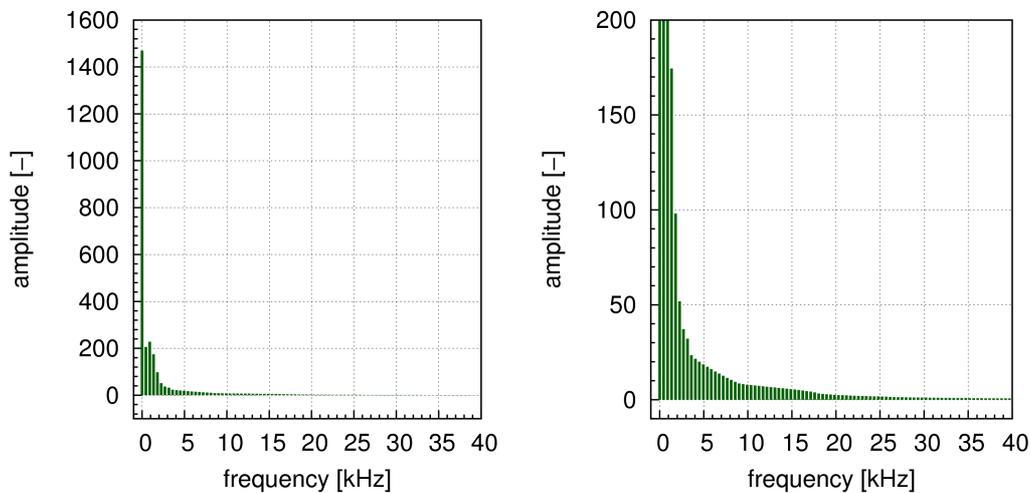
*Fig. 3.5 Ideal frequency spectrum of a force signal:
whole spectrum (l), zoomed (r)*

For the first method two points from the original spectrum around the peak of the natural frequency are picked and a straight line is calculated between them. All amplitude values for the frequencies between these two are replaced by a calculated value on the line. The linear fit method also starts with picking two points around the peak. But then a linear fit based on the amplitude between the two picked frequencies is calculated. The new amplitude values between the two frequencies are replaced by a value on the linear fit.
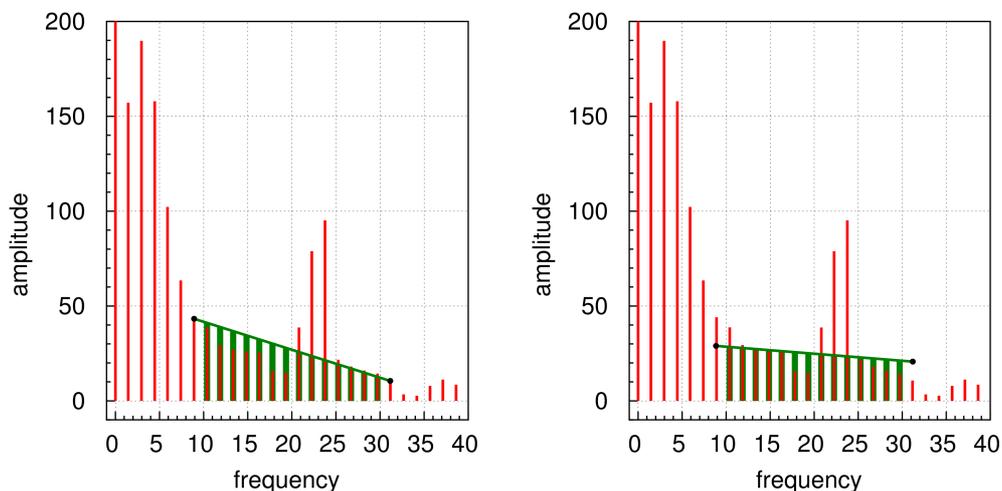
*Fig. 3.6 Smoothing of the frequency spectrum:
2-point line (l) and linear fit (r)*

After this smoothing of frequency spectrum an inverse DFT of the modified spectrum is done to get the real force signal. In Fig. 3.7 the result using a 2-point line filter is shown. The force increase at the beginning and the decrease at the end. This behaviour were not changed by the filter. The yield-drop after the yield stress appears clearly and the area below the original and the filtered curve is the same.

PC – 6 m/s
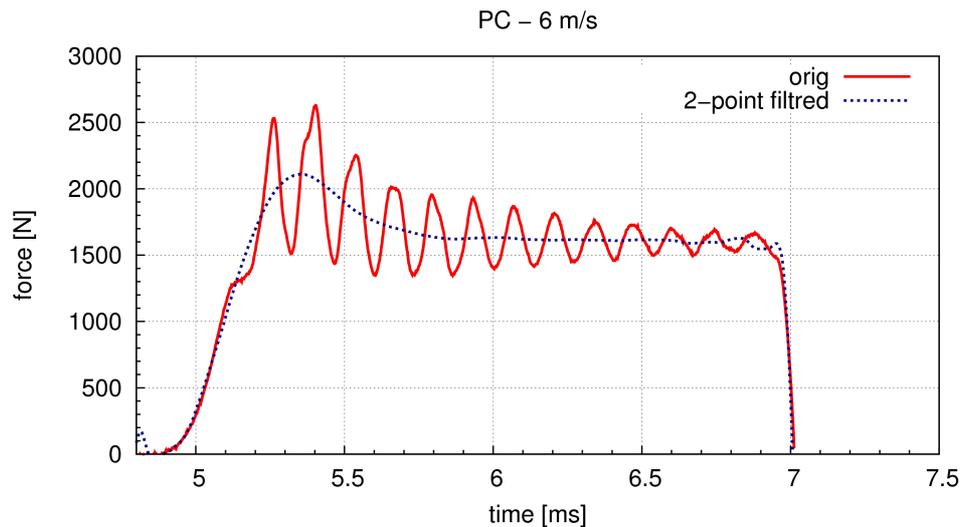
*Fig. 3.7 Original and filtered force signal for PC tested at 6 m/s using
a 2-point line filter*

4   Conclusion

The characterisation of a thermoplastic material at high strain rates needs a higher effort to get high quality results. The used model has to be more complex to include all material phenomena. So models of a higher order have to be used and therefore more parameters are introduced. Also the one-dimensional approach has to be extended to fit with the experimental data.

The complexity of the model needs a big range of experimental results from different stress states. Because of the high testing speed and the oscillations it is not possible to use the raw data for the material models without a prior signal processing. The new introduced filter method can be used to remove the superposed oscillations without changing much the of force signal in the specimen.

5   References

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