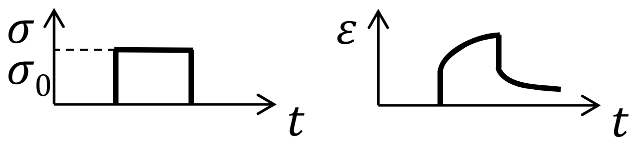
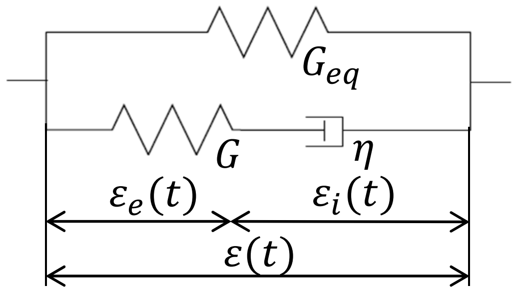
Modelling of viscoelastic materials with LS-Dyna





11th German LS-Dyna Forum 2012, Ulm

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Overview

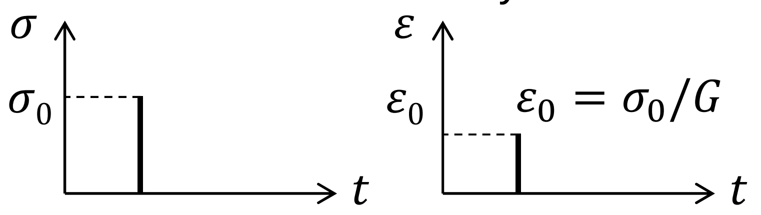
- **Motivation**

- **Models for viscoelastic materials in LS-Dyna**
  - Generalized Maxwell Model
  - Tabulated hyperelasticity

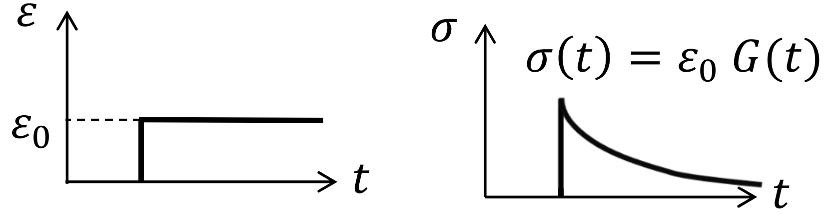
- **Modelling of a rubber material**
  - MAT_SIMPLIFIED_RUBBER
  - MAT_OGDEN_RUBBER
  - BioRID Jacket Certification Test

Motivation

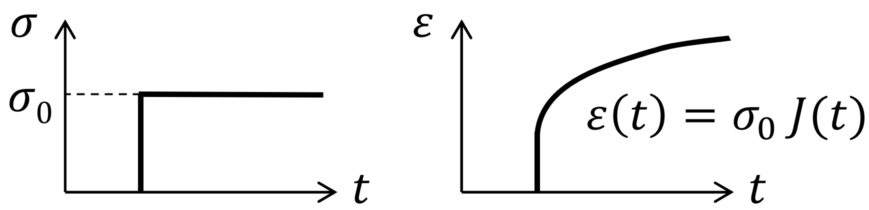
- Characteristic properties of viscoelastic solids
  - Instantaneous elasticity



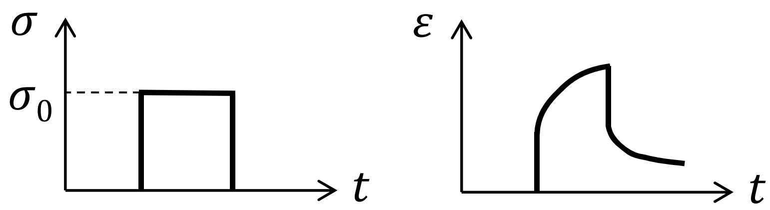
  - Stress relaxation under constant strain



  - Creep under constant stress



  - Instantaneous and delayed recovery

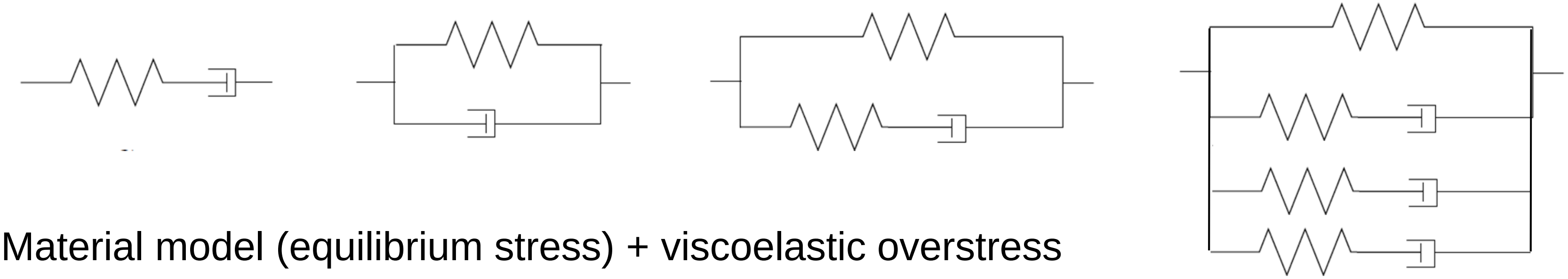


- Many materials show viscoelastic characteristics
  - Rubbers
  - Foams
  - Thermoplastics
  - Composites
  - …

Models for viscoelastic materials in LS-Dyna

- Linear viscoelastic material models based on rheological models
  - Material models: 6, 61, 76, 86, 134, 164, 234, 276,…

Maxwell-Element | Kelvin-Voigt-Element | Standard linear Solid | Generalized Maxwell Element



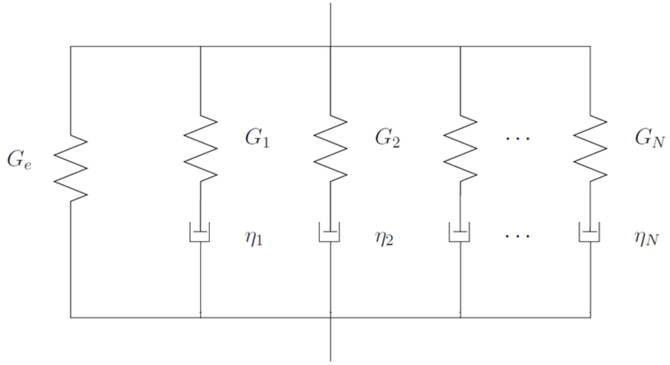
- Material model (equilibrium stress) + viscoelastic overstress
  - Material models: Hyperelasticity, (Visco-) Plasticity, …

    57, 73, 77, 87, 91, 124, 127, 129, 155, 158,175, 178,…
  - Viscoelastic Overstress: Generalized Maxwell Element

- Material models with elastic and strain rate dependent characteristics
  - Rubbers and Foams: MAT_181 (MAT_SIMPLIFIED_RUBBER/FOAM), MAT_083
  - Creep: MAT_115, MAT_188
  - Quasilinear viscoelasticity: MAT_176

Generalized Maxwell Element

- Rheological model with linear elements



Spring (Hooke) $\sigma(t) = G\varepsilon(t)$

Dashpot (Newton) $\sigma(t) = \eta \frac{\partial \varepsilon(t)}{\partial t}$

- Linear differential equation or integral equation

$$\sum_{n=0}^{N} u_n \frac{\partial^n \sigma(t)}{\partial t^n} = \sum_{m=0}^{N} q_m \frac{\partial^m \varepsilon(t)}{\partial t^m}$$

u_n, q_m : material constants

$$\sigma(t) = \int_0^t G(t-u) \frac{\partial \varepsilon(u)}{\partial u} du$$

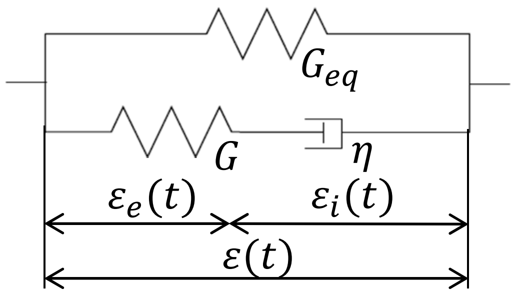
$G(t)$: relaxation function

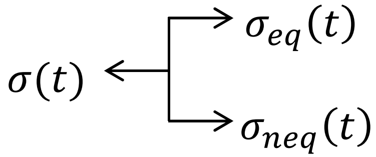
- 3D-Formulation

$$\sigma_{ij}(t) = \int_0^t 2G(t-u) \left(\frac{\partial \varepsilon_{ij}(u)}{\partial u} - \frac{1}{3} \frac{\partial \varepsilon_{kk}(u)}{\partial u} \delta_{ij} \right) du + \int_0^t K(t-u) \frac{\partial \varepsilon_{kk}(u)}{\partial u} \delta_{ij} du$$

Generalized Maxwell Element

- $N = 1$: Standard linear solid





$$\sigma_{eq}(t) = G_{eq}\, \varepsilon(t)$$

$$\sigma_{neq}(t) = G\, \varepsilon_e(t) = \eta \frac{\partial \varepsilon_i(t)}{\partial t}$$

$$\sigma(t) = \sigma_{eq}(t) + \sigma_{neq}(t)$$

$$\varepsilon(t) = \varepsilon_e(t) + \varepsilon_i(t)$$

$$\frac{\eta}{G}\sigma(t) + \frac{\partial \sigma(t)}{\partial t} = \frac{G_{eq}}{G}\eta\varepsilon(t) + (G + G_{eq})\frac{\partial \varepsilon(t)}{\partial t}$$

Linear differential equation

1. Solution of the homogeneous differential equation $\sigma_h(t) = e^{\lambda t}$

$$\frac{\eta}{G}\sigma_h(t) + \frac{\partial \sigma_h(t)}{\partial t} = 0 \qquad \sigma_h(t) = e^{-\frac{\eta}{G}t} = e^{-\beta t} \qquad \beta = \frac{\eta}{G}$$: decay constant

2. Particular solution: Variation of constants $\sigma(t) = k(t)e^{-\beta t}$

$$\sigma(t) = \int_0^t \left(G_{eq} + G\, e^{-\beta\,(t-u)} \right) \frac{\partial \varepsilon(u)}{\partial u} du = \int_0^t G(t-u) \frac{\partial \varepsilon(u)}{\partial u} du$$

Integral equation

Generalized Maxwell Element

- $N \geq 1:$ Prony-Series

$$G(t) = G_{eq} + \sum_{i=1}^{N} G_i e^{-\beta_i t} \qquad \sigma(t) = \int_0^t G(t-u)\frac{\partial \varepsilon(u)}{\partial u} du$$

- Linear viscoelasticity for simple, homogeneous and non-aging materials

$$\sigma(t) = \mathcal{F}_{u=0}^{\infty}(\varepsilon(t-u))$$

  - Stress-Strain Linearity

$$\mathcal{F}_{u=0}^{\infty}\big(\boldsymbol{\alpha}\varepsilon(t-u)\big) = \boldsymbol{\alpha}\mathcal{F}_{u=0}^{\infty}\big(\varepsilon(t-u)\big) = \boldsymbol{\alpha}\sigma(t)$$

„An increase in the stimulus by an arbitrary factor $\boldsymbol{\alpha}$ must increas the response by the same factor.“

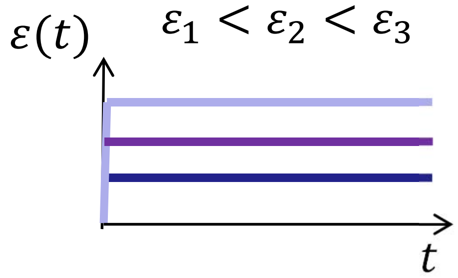
  - Boltzmann Superposition Principle

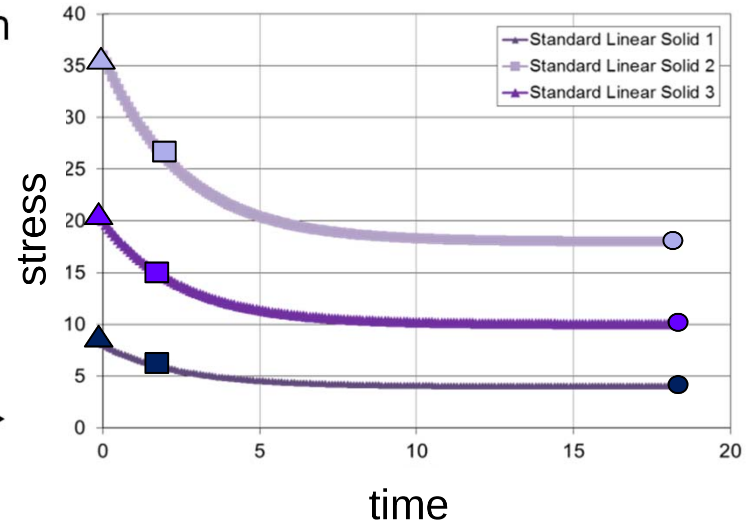
$$\mathcal{F}_{u=0}^{\infty}\left(\sum_{n=1}^{\infty} \varepsilon_n(t-u)\right) = \sum_{n=1}^{\infty} \mathcal{F}_{u=0}^{\infty}(\varepsilon_n(t-u))$$

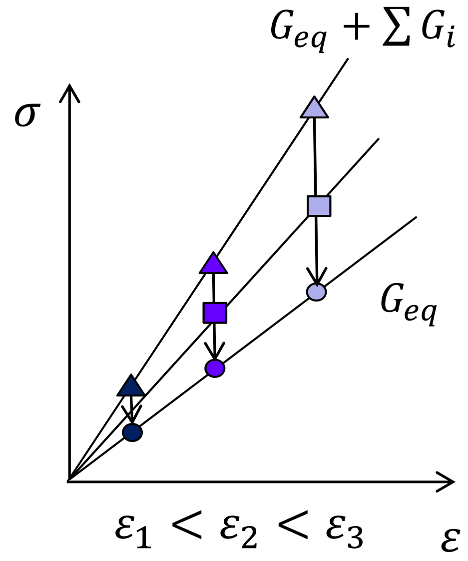
„An arbitrary sequence of stimuli must elicit a response which is equal to the sum of the responses which would have been obtained if all stimuli had acted independently.“

Generalized Maxwell Element

- Strain Step Function







$$\varepsilon_1(t) = \varepsilon_1 \mathcal{H}(t) \qquad \sigma_1(\mathrm{t}) = \varepsilon_1[G_{eq} + \sum G_i\, e^{-\beta_i t}]$$

Linear equilibrium elasticity G_{eq}

- Fading memory $\lim_{t \to \infty} \sigma_1(\mathrm{t}) = \varepsilon_1 G_{eq}$

- Stress-Strain Linearity

$$\varepsilon_2(t) = \varepsilon_2 \mathcal{H}(t) = \frac{\varepsilon_2}{\varepsilon_1} \varepsilon_1 \mathcal{H}(t) = \alpha \varepsilon_1(t)$$

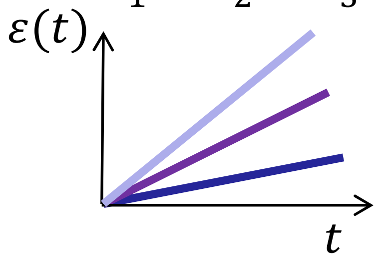
$$\sigma_2(t) = \alpha\, \sigma_1(t)$$

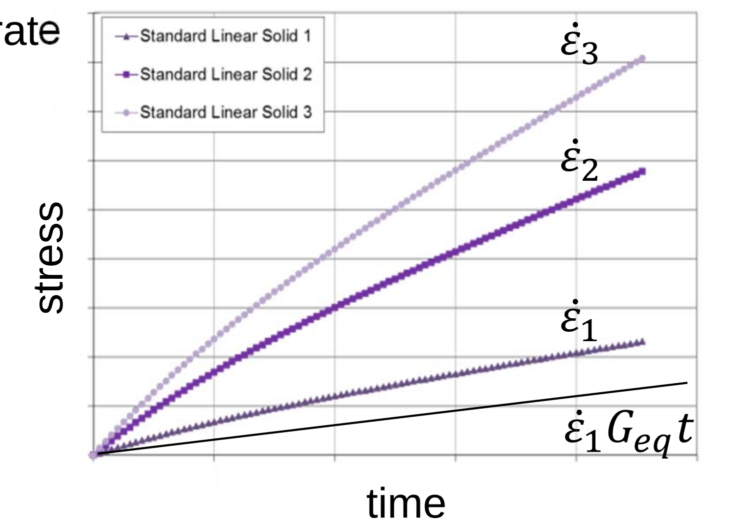
One test curve defines all other curves.
Linear viscoelastic material behaviour cannot be identified with one relaxation curve.

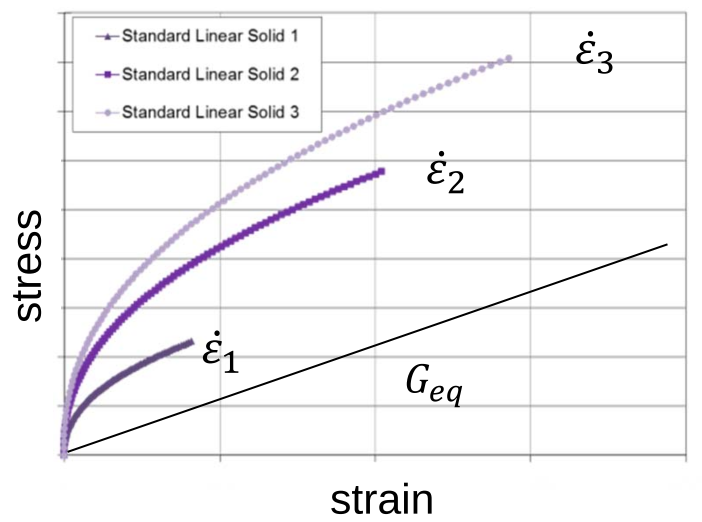
Generalized Maxwell Element

- Constant strain rate

$\dot{\varepsilon}_1 < \dot{\varepsilon}_2 < \dot{\varepsilon}_3$







$$\varepsilon_1(t) = \dot{\varepsilon}_1 t$$

$$\sigma_1(t) = \dot{\varepsilon}_1 G_{eq} t + \dot{\varepsilon}_1 \sum \frac{G_i}{\beta_i} \left(1 - e^{-\beta_i t}\right)$$

$$\sigma_1(\varepsilon) = G_{eq} \varepsilon + \dot{\varepsilon}_1 \sum \frac{G_i}{\beta_i} \left(1 - e^{-\beta_i \frac{\varepsilon}{\dot{\varepsilon}_1}}\right)$$

$$\lim_{t \to \infty} \sigma_1(t) = \dot{\varepsilon}_1 G_{eq} t + \dot{\varepsilon}_1 \sum \frac{G_i}{\beta_i}$$

$$\lim_{\varepsilon \to \infty} \sigma_1(\varepsilon) = G_{eq} \varepsilon + \dot{\varepsilon}_1 \sum \frac{G_i}{\beta_i}$$

- Stress-Strain Linearity

$$\varepsilon_2(t) = \dot{\varepsilon}_2 \, t = \frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1} \dot{\varepsilon}_1 t = \alpha \varepsilon_1(t)$$

$$\sigma_2(t) = \alpha \, \sigma_1(t)$$

Linear viscoelasticity:
Nonlinear stress-strain curves
at constant strain rate

Generalized Maxwell Element

- Equilibrium and non-equilibrium stress

$$\sigma(t) = \mathcal{F}_{u=0}^{\infty}(\varepsilon(t-u))$$

$$\sigma(t) = \int_0^t G(t-u)\frac{\partial\varepsilon(u)}{\partial u}du$$

$$G(t) = G_{eq} + \sum_{i=1}^{N} G_i e^{-\beta_i t}$$

$\longrightarrow$

$$\sigma(t) = \mathcal{H}(\varepsilon(t)) + \widetilde{\mathcal{H}}_{u=0}^{\infty}(\varepsilon(t-u))$$

$$\sigma(t) = G_{eq}\varepsilon(t) + \int_0^t \tilde{G}(t-u)\frac{\partial\varepsilon(u)}{\partial u}du$$

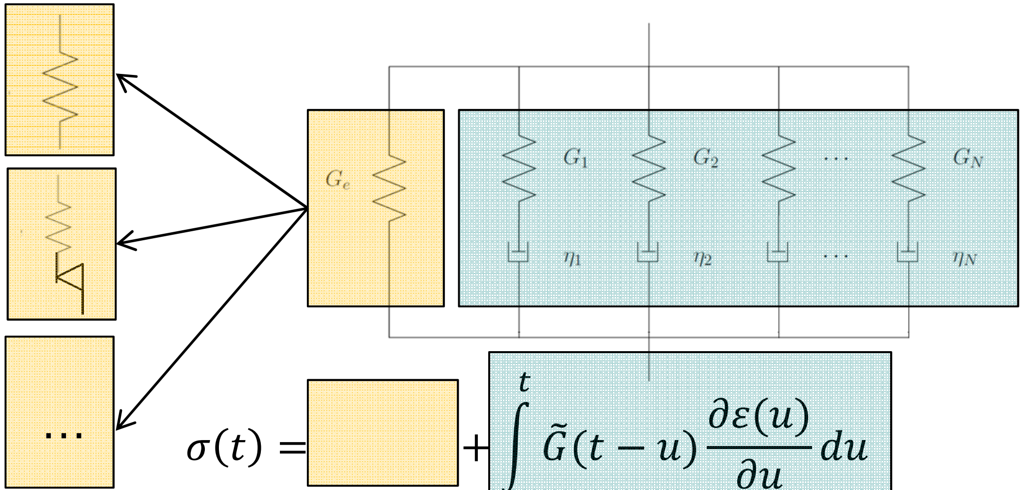
$$\tilde{G}(t) = \sum_{i=1}^{N} G_i e^{-\beta_i t}$$

- Non-equilibrium, viscoelastic stress as overstress for different material models

Nonlinear spring
e. g. hyperelasticity
MAT_077_O

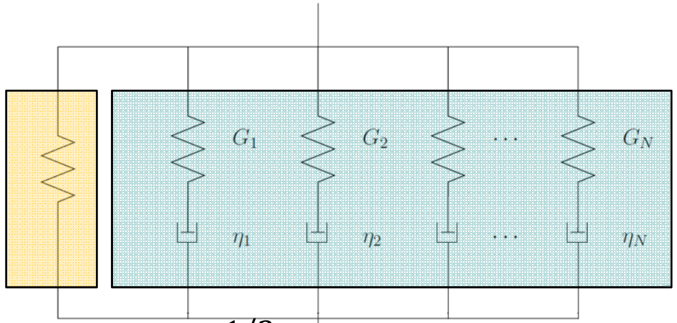
Friction element
e. g. Elasto-Plasticity

Viscoplasticity,
Damage, Failure,…



$$\sigma(t) = \boxed{} + \int_0^t \tilde{G}(t-u)\frac{\partial\varepsilon(u)}{\partial u}du$$

MAT_OGDEN_RUBBER



- Equilibrium stress: Hyperelasticity
    - Ogden strain energy potential and principal stresses

$$W = \sum_{i=1}^{3}\sum_{j=1}^{m}\frac{\mu_j}{\alpha_j}\left(\lambda_i^{*\alpha_j} - 1\right) + \mathrm{K}(J - 1 - \ln J)$$

$$\sigma_i = \frac{1}{\lambda_k\lambda_j}\frac{\partial W}{\partial \lambda_i} = \sum_{j=1}^{m}\frac{\mu_i}{J}\left[\lambda_i^{*\alpha_j} - \sum_{k=1}^{3}\frac{\lambda_k^{*\alpha_j}}{3}\right] + K\frac{J-1}{J}$$

$$\lambda_i^* = J^{-1/3}\lambda_i$$

λ_i : principal stretches
μ_j, α_j: material constants

- Uniaxial loading for incompressible material: $J \approx 1, \lambda_2 = \lambda_3 = (\lambda_1)^{-1/2} = (\lambda_{uni})^{-1/2}$

$$\sigma_{uni} = \sum_{j=1}^{m}\left(\mu_j\lambda_{uni}^{\alpha_j} - \mu_j\lambda_{uni}^{-\frac{1}{2}\alpha_j}\right) = \sum_{j=1}^{m}\left(\mu_j(1+\varepsilon_{uni})^{\alpha_j} - \mu_j(1+\varepsilon_{uni})^{-\frac{1}{2}\alpha_j}\right)$$

- Non-equilibrium stress: Viscoelasticity
  Generalized Maxwell Element for deviatoric deformation

$$\tilde{G}(t) = \sum_{i=1}^{N} G_i e^{-\beta_i t} \overset{\nu=0.5}{\triangleq} \sum_{i=1}^{N}\frac{E_i}{3}e^{-\beta_i t}$$

Tabulated hyperelasticity

- Hyperelasticity without parameter identification
  - MAT_FU_CHANG_FOAM (MAT_083)
  - MAT_SIMPLIFIED_RUBBER (MAT_181)

- Ogden Model: Series expansion for incompressible material

$$f(\lambda_i^*) = \sum_{j=1}^{m} \mu_j \lambda_i^{*\alpha_j}$$

$$\sigma_i = \sum_{j=1}^{m} \frac{\mu_i}{J}\left[\lambda_i^{*\alpha_j} - \sum_{k=1}^{3} \frac{\lambda_k^{*\alpha_j}}{3}\right] + K\frac{J-1}{J} = \frac{1}{J}\left[f(\lambda_i^*) - \frac{1}{3}\sum_{k=1}^{3} f(\lambda_i^*)\right] + K\frac{J-1}{J}$$

$$\sigma_{uni} = \sum_{j=1}^{m}\left(\mu_j \lambda_{uni}^{\alpha_j} - \mu_j \lambda_{uni}^{-\frac{1}{2}\alpha_j}\right)$$

$$f(\lambda_i) = \sum_{j=1}^{m} \mu_j {\lambda_i}^{\alpha_j} = \sigma_{uni}(\lambda_i) + \sum_{j=1}^{m} \mu_j {\lambda_i}^{-\frac{1}{2}\alpha_j} = \sigma_{uni}(\lambda_i) + \sigma_{uni}\left({\lambda_i}^{-\frac{1}{2}}\right) + \sum_{j=1}^{m} \mu_j {\lambda_i}^{\frac{1}{4}\alpha_j}$$

$$= \sigma_{uni}(\lambda_i) + \sigma_{uni}\left({\lambda_i}^{-\frac{1}{2}}\right) + \sigma_{uni}\left(\lambda_i^{\frac{1}{4}}\right) + \sum_{j=1}^{m} \mu_j {\lambda_i}^{-\frac{1}{8}\alpha_j}$$

$$= \sum_{n=1}^{\infty} \sigma_{uni}\left(\lambda_i^{\left(-\frac{1}{2}\right)^n}\right)$$

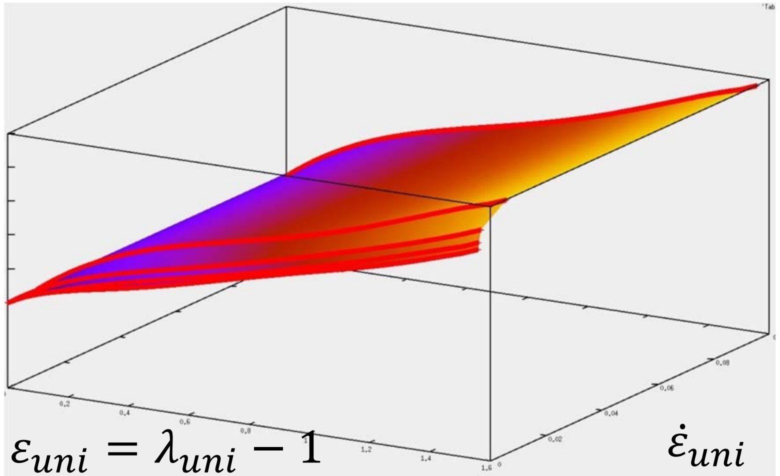
Exit if $\left\|\lambda_i^{\left(-\frac{1}{2}\right)^n}\right\| \le 1.01$

Principal stress can directly be calculated from uniaxial stress-strain curve.

Tabulated hyperelasticity

- Rate dependency
  - Loading: Interpolation between strain rates within table definition
  - Unloading: lowest strain rate or damage model

σ_{uni}



- Damage model
  - Closed loading and unloading path define damage evolution $d = d(\varepsilon_{uni})$
  - Unloading: $\sigma_{uni} = \sigma_{uni}(1 - d)$

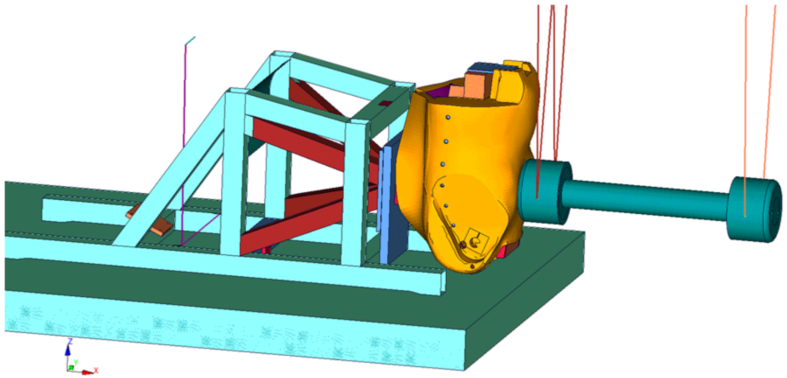
Modelling of a rubber material

- Silicone – Jacket in BIORID II

- Material characterization
  - incompressible material
  - compression and tensile tests
  - quasistatic and dynamic testing

- Modelling with LS-Dyna: Comparison of strain rate dependent modelling
  - MAT_SIMPLIFIED_RUBBER (MAT181)
  - MAT_OGDEN_RUBBER (MAT077_O)

- BioRID Jacket Certification Test
  - Pendulum acceleration
  - Sled acceleration

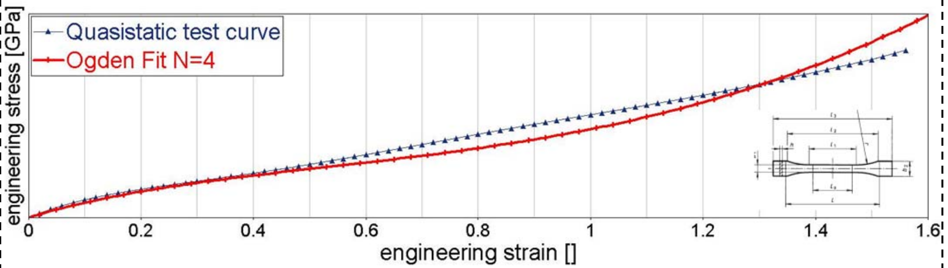


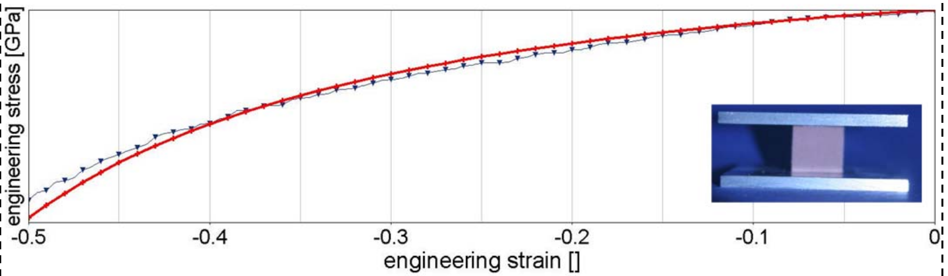
Material characterization

- Equilibrium stress

  Quasistatic test curves –
  Fit to Ogden strain energy potential
  for MAT077 and MAT181

$$\sigma_i = \sum_{j=1}^{N} \frac{\mu_i}{J} \left[\lambda_i^{*\alpha_j} - \sum_{k=1}^{3} \frac{\lambda_k^{*\alpha_j}}{3} \right] + K \frac{J-1}{J}$$



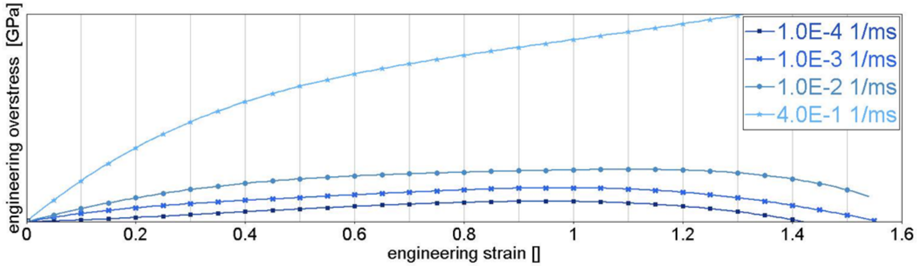


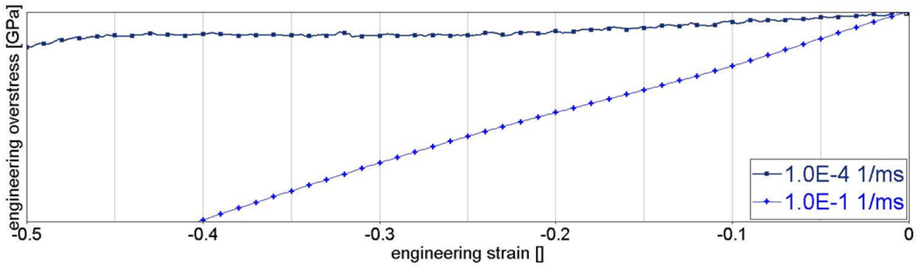
- Non-equilibrium stress

  Viscoelastic overstress
  = test curves – Ogden-Fit

- MAT181: Strain-rate dependent table

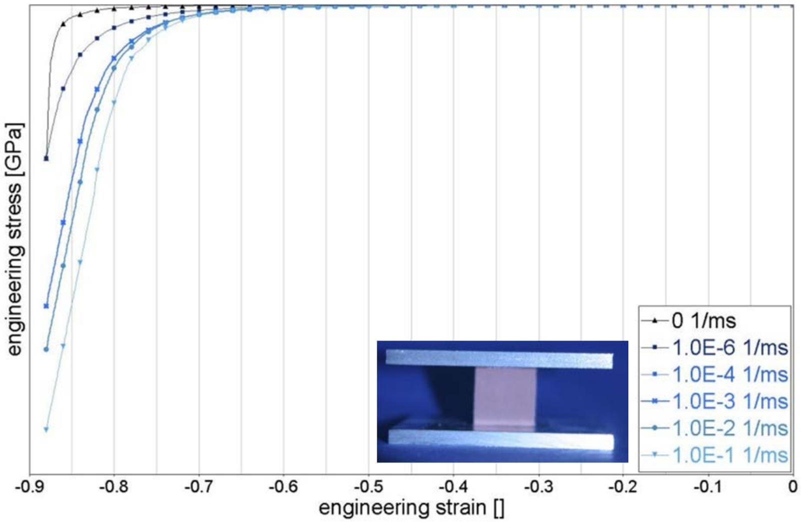
- MAT077: Generalized Maxwell Element

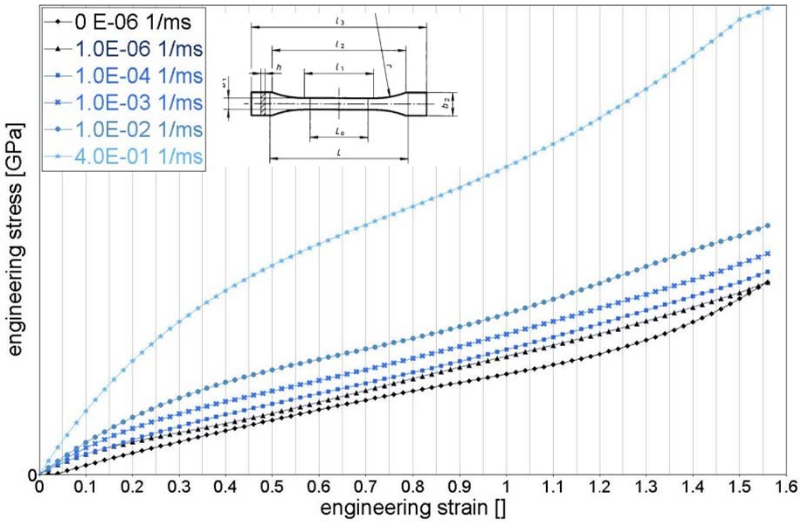




MAT_SIMPLIFIED_RUBBER

- Table: Ogden-Fit and dynamic test curves

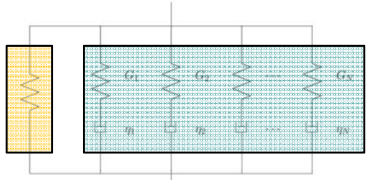




- Strain rate dependency
  - engineering strain rate (RTYPE=1)
  - simple average of 12 time steps (AVGOPT=0)
  - rate effects are treated identically in tension and compression (TENSION=1)

- Unloading
  - Internal damage formulation based on quasistatic unloading path (LCUNLD)
  - Rate dependent unloading path

MAT_OGDEN_RUBBER

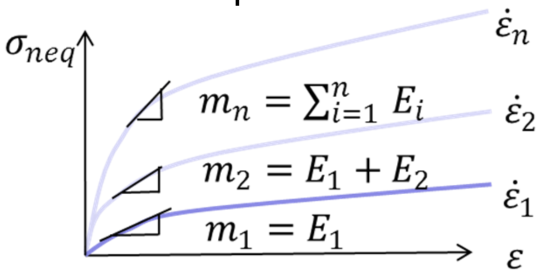


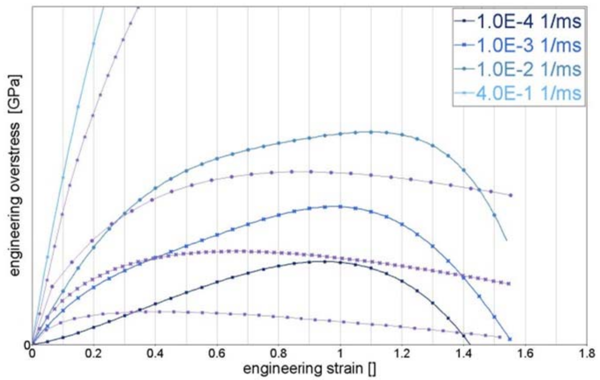
- Parameter identification for Prony-Series

$$\tilde{G}(t) = \sum_{i=1}^{N} G_i e^{-\beta_i t} \overset{\nu=0.5}{\triangleq} \sum_{i=1}^{N} \frac{E_i}{3} e^{-\beta_i t}$$

- uniaxial tensile tests ($\dot{\varepsilon}_i = cst.$)
- limited to relevant time scales $\beta_i = \frac{1}{\dot{\varepsilon}_i}$

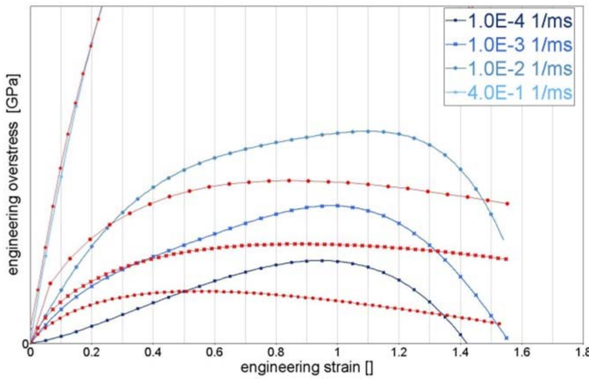
Slope method



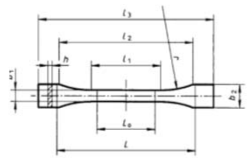


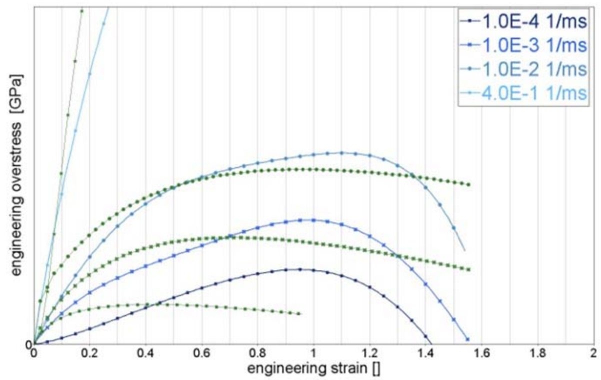
Fit of analytical answer to test curves

$$\sigma_{neq}(\varepsilon, \dot{\varepsilon}) = \dot{\varepsilon} \sum_{i=1}^{N} E_i \frac{1}{\dot{\beta}_i} \left(1 - e^{-\frac{\dot{\beta}_i}{\dot{\varepsilon}}\varepsilon}\right)$$

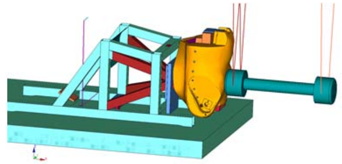


Simulation of material tests and optimization

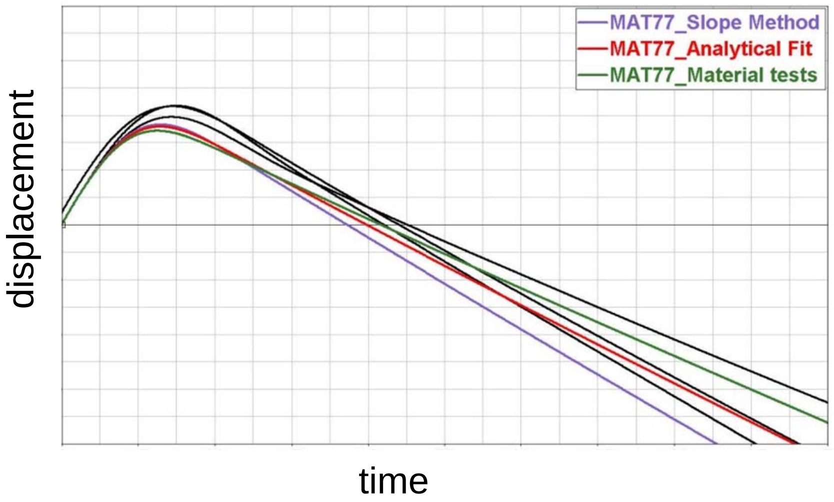




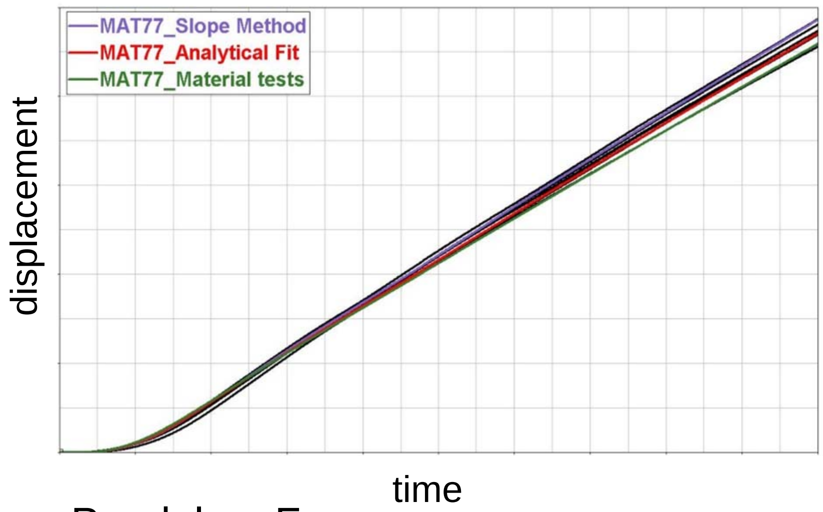
BioRID Jacket Certification Test: MAT_77



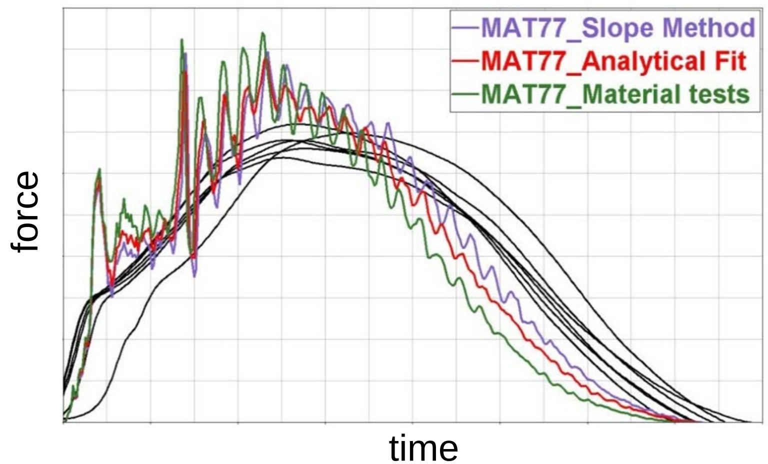
- Pendulum Displacement



- Sled Displacement



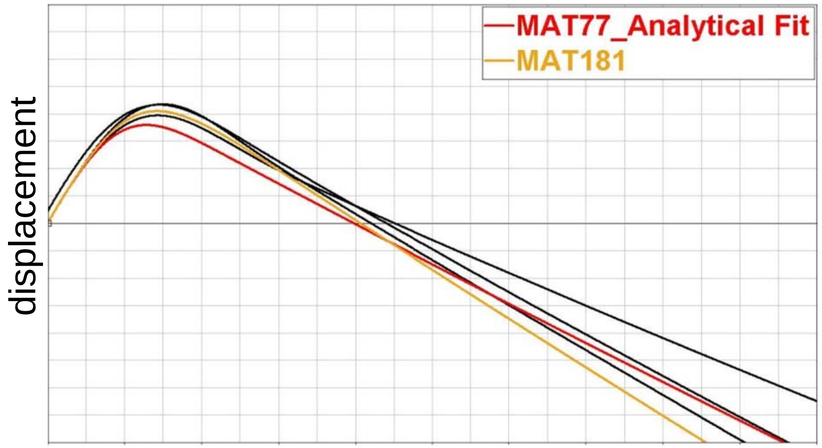
- Pendulum Force



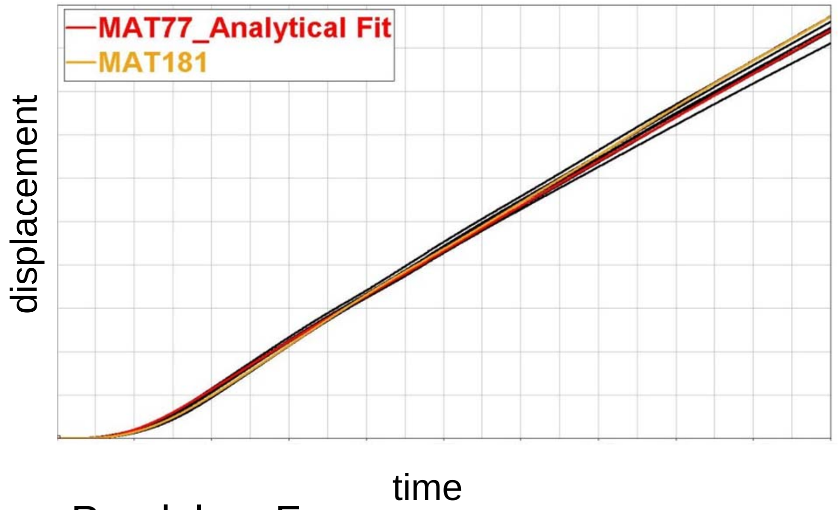
- Minor differences for 3 Prony series
- Initial stiffness:
  turning point and pendulum force
- Best result for analytical fit

BioRID Jacket Certification Test: MAT_77 vs. MAT_181

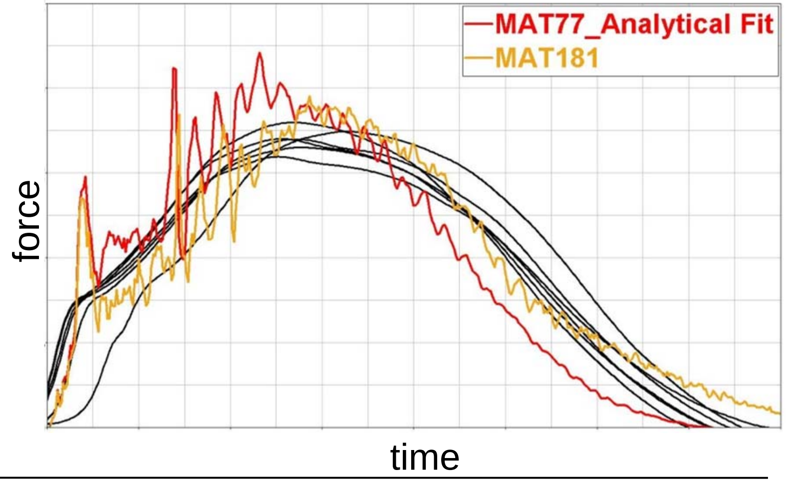
- Pendulum Displacement



- Sled Displacement



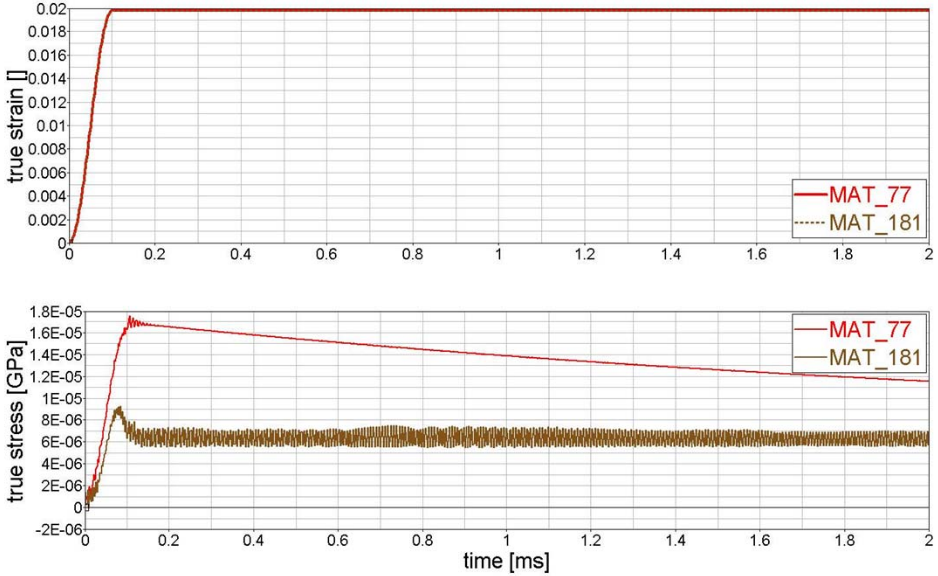
- Pendulum Force



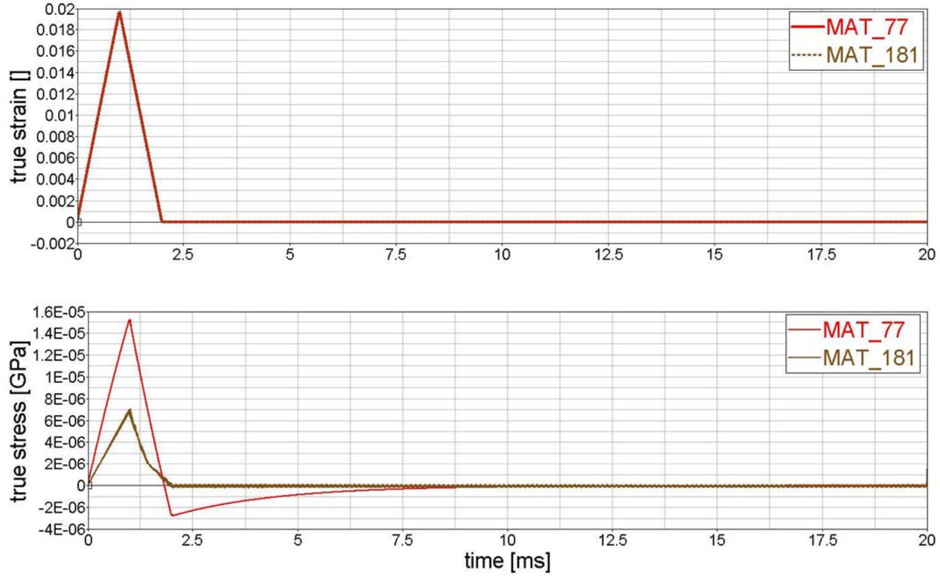
- MAT_181
  - Better agreement for loading
  - Too small hysteresis

Tabulated Hyperelasticity vs. Linear Viscoelasticity

- Relaxation

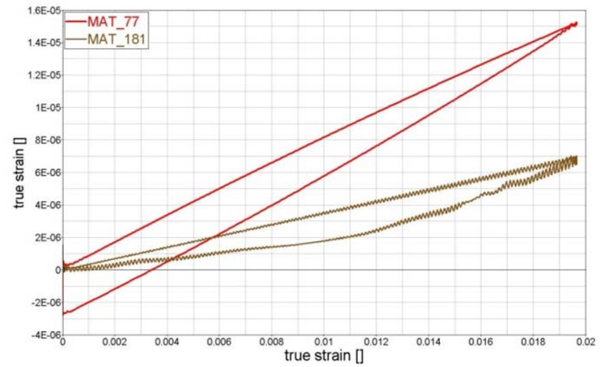


- Loading and Unloading



- MAT_181: „Relaxation“ due to averaging over last 12 time steps

- MAT_77: longer relaxation time leads to sign reversal in the stress upon unloading



Summary

- Rheological models
  - Linear viscoelasticity
  - No difference between compression and tension
  - Time-consuming parameter identification
  - Test curves differ from model answer

- Tabulated hyperelasticity
  - Nonlinear rate dependency in compression and tension
  - Direct tabulated input using test results
  - Limited modelling of characteristic viscoelastic properties: no creep, no relaxation
  - Difficult to match unloading behaviour in component test simulations

Thank you for your attention!