

# Decision Making in Multi-Objective Optimization for Industrial Applications - Data Mining and Visualization of Pareto Data

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## Summary:

Optimization of engineering structures where multiple (more or less conflicting) objectives are simultaneously considered, is getting more and more attractive in automotive industries. They usually involve a large number of design variables and the objectives are subject to certain constraints. Unlike single-objective problems, there are many trade-off solutions.

The most common approach of using a single aggregate objective function (AOF), though simple, is not appropriate in most cases because a) it requires a priori information e.g., weights associated with each objective for weighted linear sum of the objectives method, that might not be available; and b) this approach yields a single trade-off solution instead of all possible trade-off solutions.

Multi-objective evolutionary algorithms (MOEA) seem to be the best choice at the moment to overcome these issues. A set of solutions (Pareto data) is obtained as result, which reflect distinct trade-off solutions. A (optimal) decision needs to be taken to choose the most suitable trade-off among multiple conflicting objectives.

Data mining and visualization techniques for high dimensional data provide helpful information to substantially augment the decision making (alternative design selection) in multi-objective optimization environment. A graphical approach to visualize the Pareto frontier is an intuitive and suitable approach to investigate the trade-off for three or fewer dimensions (objectives). However, it is not trivial to study relations in higher dimensions hence many visualization methods are proposed. The basic idea of these techniques is to reduce the dimensionality without losing the relevant information required to recognize and understand relations and characteristics of the high dimensional Pareto data. Among the several developments in these fields, the Parallel Coordinates Plot (PCP), the Hyper-Radial Visualization (HRV), and the Self Organizing Maps (SOM) have been found the most promising.

The parallel coordinates plot assigns one axis to each dimension and many dimensions are aligned in parallel. A data point is represented as a line connecting different axes. The HRV is based on a radial calculation and transfers the multi-dimensional data to a two-dimensional data set by grouping the weighted objectives, that leads to a final solution with respect to the selected weights and the grouping. The designer incorporates his preferences by modifying the selection. The SOM algorithm projects the multi-dimensional Pareto data onto a two-dimensional map, whereby similar data is mapped to neighboring locations on the map. The lattices are color-coded to show the variation of the data on the map.

The concepts of PCP, HRV and SOM are explained along with the various forms of visualization of Pareto data. All three approaches are investigated and respective pros and cons are identified using a shape optimization case crash application executed with LS-OPT. An implementation in the data mining and visualization framework D-SPEX is also provided.

## Keywords:

Multi-Objective Optimization (MOO), LS-OPT, Visualization, Parallel Coordinates, SOM, HRV

## 1 Introduction

Most engineering optimization problems require multi-objective optimization that have no unique optimum because the objectives usually conflict. Hence multi-objective evolutionary algorithms (MOEA) are used to solve the optimization problem. Common visualization techniques have capabilities to visualize three to five dimensions. For multi-objective optimizations, not only the relationship between objectives is of interest, but also the relationship between variables and objectives, so the limit of five dimensions is usually exceeded. New visualization methods have to be investigated to help the engineer exploring the Pareto optimal solutions and finally determining his favorite design.

In the following sections, the example problem is presented, and the visualization methods are explained and compared using the example problem.

## 2 Example Problem

This optimization study is carried out by considering a crashworthiness simulation of a National Highway Transportation and Safety Association (NHTSA) vehicle undergoing a full frontal impact. The finite element model for the full vehicle, containing approximately 54,800 elements (obtained from NCAC website [1]), is shown in Figure 1. The LS-DYNA [2] explicit solver is used to simulate the crash.

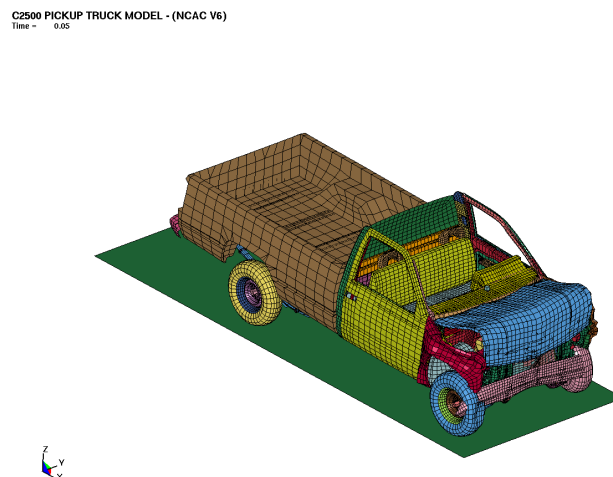


Figure 1: Finite element crash model of a pickup truck

Table 1: Baseline design and bounds on design variables

Variable description	Name	Lower bound	Baseline design	Upper bound
Rail front-right-inner	T1	2.50	3.137	3.765
Rail front-right-outer	T2	2.48	3.112	3.750
Rail front-left-inner	T3	2.40	2.997	3.600
Rail front-left-outer	T4	2.40	3.072	3.600
Rail right-back	T5	2.72	3.400	4.080
Rail left-back	T6	2.85	3.561	4.270
Bumper	T10	2.16	2.700	3.240
Radiator bottom	T64	1.00	1.262	1.510
Cabin bottom	T73	1.60	1.990	2.400

The gauges of structural components in the vehicle are parameterized directly in the solver input file. Nine gauge thicknesses associated with front-right-inner, front-right-outer, front-left-inner, front-left-outer, back-left and back-right rails, bumper, bottom-under radiator MTG, and bottom-center cabin member,

are taken as design variables. The parts affected by the design variables are shown in Figure 2. The range of these design variables is chosen as within +/-20% of the baseline design variable values. The baseline design and the bounds on the variables are given in Table 1.

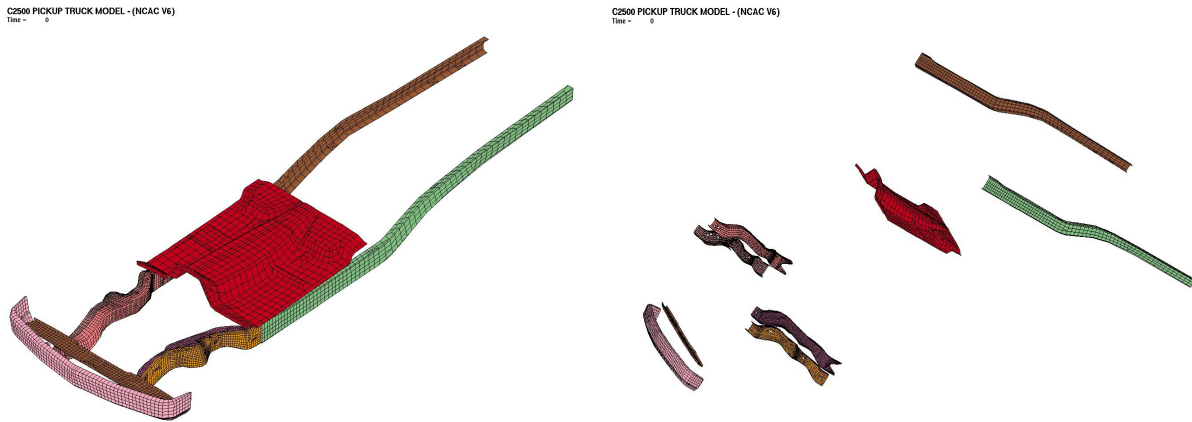


Figure 2: Thickness design variables (with exploded view)

The crash performance of the vehicle is characterized by considering the maximum acceleration, maximum displacement that links to intrusion, time taken by the vehicle to reach zero velocity state, and different stage pulses. These responses are taken at the accelerometer mounted in the middle of the front seat. To reduce the influence of numerical noise, filtered acceleration is considered and different entities are averaged over two accelerometer nodes. While constraints are imposed on some of these crash performance criteria like stage pulses, it is desirable to optimize the performance with respect to other criteria. Thus a multi-objective optimization problem can be formulated as follows:

- minimize
  - mass and peak acceleration,
- maximize
  - time-to-zero-velocity and maximum displacement,
- subject to
  - the constraints on variables and performance.

The design variable bounds are given in Table 1 and the performance constraints, namely maximum displacements and stage pulses, are specified in Table 2.

Table 2: Design constraints

	Lower bound	Upper bound
Maximum displacement ( $x_{\text{crash}}$ )	-	721mm
Stage 1 pulse ( $x_{\text{crash}}$ )	-	7.48g
Stage 2 pulse ( $x_{\text{crash}}$ )	-	20.20g
Stage 3 pulse ( $x_{\text{crash}}$ )	-	24.50g

The three stage pulses are calculated from the averaged SAE filtered (60Hz) acceleration and displacement of the accelerometer nodes in the following fashion

$$\text{Stage } j \text{ pulse} = -\frac{k}{(d_2 - d_1)} \int_{d_1}^{d_2} \ddot{x} dx \quad (1)$$

with  $k = 0.5$  for  $j = 1$  and  $k = 1.0$  otherwise.

The integration limits  $(d_1; d_2) = (0; 200); (200; 400); (400; \text{Max}(\text{displacement}))$  for  $j = 1, 2, 3$  respectively, represent different structural crash events. All displacement units are mm and the minus sign is used to convert acceleration to deceleration. All objectives and constraints are scaled to avoid dimensionality issues.

Each LS-DYNA simulation for this example problem takes approximately 3.5 hrs on Intel Xeon 2.66 GHz processor machine with 4GB memory. Since this is a computationally intensive problem, a metamodel-based multi-objective optimization method (implemented in LS-OPT [3]) is used to obtain candidate Pareto optimal set. For all responses, the RBF metamodels were constructed using a PRESS based selection criterion as recommended by Goel and Stander [4]. A total of 997 simulation points obtained by evolving a population of 20 individuals for 50 generations (Goel et al. [3]) were used to construct metamodels.

The elitist non-dominated sorting genetic algorithm (NSGA-II) with real encoding [6] is used to find candidate Pareto optimal solutions. After evolving for 250 generations, the population of 100 individuals yielded 590 candidate Pareto optimal solutions. This set of solutions is analyzed using four visualization strategies.

## 2.1 Tradeoff Plot

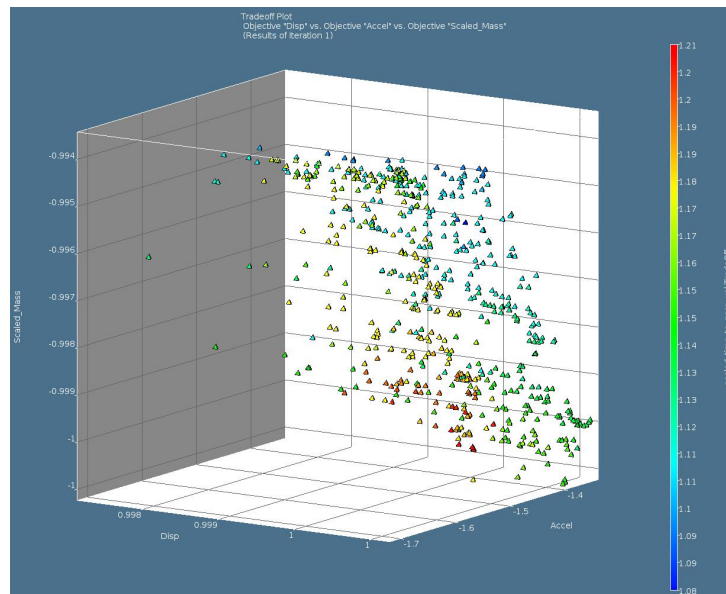


Figure 3: Trade-off among four objectives

The simple tradeoff plot (Figure 3) shows three objectives on the three axes and the fourth objective using color. One can easily visualize the conflict among objectives, infer some trade-offs like significant improvement in acceleration characteristics can be obtained by trading very small displacement. There apparently is a discontinuity in the Pareto front that might be due to the nature of the problem or just an artifact of optimization (inability to find all points). However, this graph provides limited information about the Pareto optimal front, for example, the information about the design variables and constraints is missing.

## 2.2 Parallel Coordinate Plot (PCP)

The parallel coordinate plot (Figure 4) is the simplest of all the graphs and offers more details in correlating the design variables with the responses. Each dimension is visualized on a vertical axis and each data point as a polyline connecting the respective values on the vertical axis. The number of dimensions that can be visualized using the Parallel Coordinate Plot is not restricted, but too many data points

may result in a dense and unclear view. But one can eliminate many design points by quickly changing the ranges of the variables, or objectives. Such effort helps in decision making but the most important information about the trends is missing here.

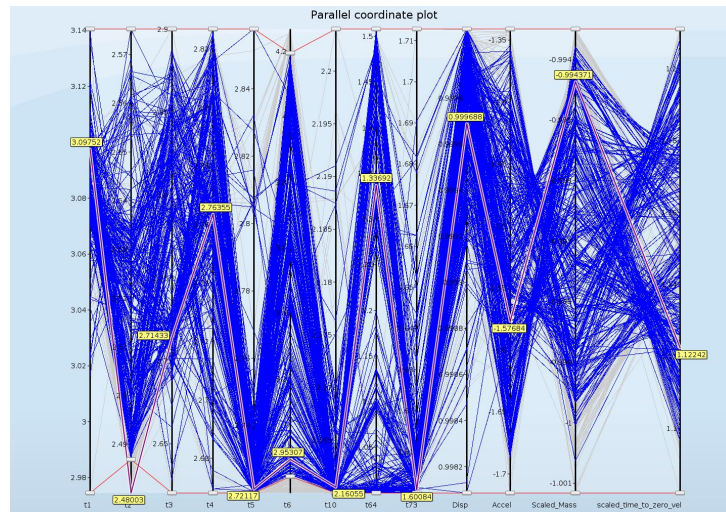


Figure 4: Parallel coordinate plot

### 2.3 Hyper-Radial Visualization (HRV)

Hyper-radial visualization is a more complex way of combining the preference structure of the designer with the objectives in such a way that multi-dimensional plots are reduced to more conventional two-dimensional graphs [7]. The normalized objectives are grouped and a weighted sum of each group is calculated. These values are displayed in two dimensions. The most prominent feature of this plot is the ability to single-out preferred designs among all the Pareto optimal solutions. In addition to the two-dimensional data set, indifference curves are displayed. Each point on an indifference curve has the same preference. The indifference curves are circles around the origin. The indifference curves closer to the origin, which is the ideal point for the normalized objectives, are of greater importance. The point on the nearest indifference curve to the ideal point is calculated, which is the optimal solution for the selected weights.

In Figure 5(a), all designs are assigned equal weight. Scaled Displacement and acceleration are plotted on the x-axis, and mass and time-to-zero-velocity are plotted on the y-axis. For this combination the best design is shown between HRV contours 0.2 and 0.3. When the weight associated with displacement is reduced to zero, the HRV plot shown in Figure 5(b), the shape of the Pareto frontier changes significantly and there are a few designs near the optimal HRV contour.

Table 3: Optimal designs with different designer preferences. All objectives are scaled values. w=1 case indicates the case when all objectives have equal weight and “no d.” case corresponds to zero weight for displacement objective and unit weight for all other objectives.

	T1	T2	T3	T4	T5	T6	T10	T64	T73	Disp.	Accel.	Mass	Time
w=1	3.07	2.50	2.88	2.76	2.73	3.46	2.16	1.20	1.60	0.998	-1.59	-0.997	1.14
no d.	3.08	2.56	2.87	2.77	2.74	4.07	2.16	1.31	1.68	1.000	-1.49	-1.000	1.15

The best designs for the two cases are given in Table 3. One can easily see that dropping the displacement objective significantly improves the acceleration objective while resulting in small penalties for other objectives.

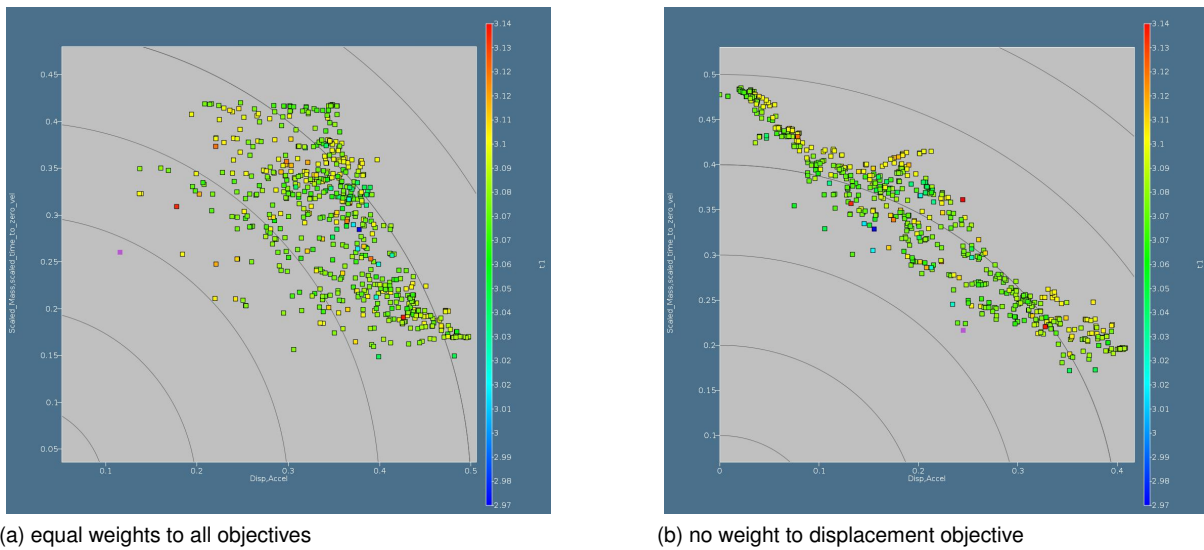


Figure 5: Hyper radial visualization

## 2.4 Self organizing maps (SOM)

Self-Organizing Map [9] is an unsupervised neural network algorithm that projects multi-dimensional data onto a two-dimensional array of nodes. Each node is associated with a randomly initialized  $n$ -dimensional weight vector, where  $n$  is the dimension of the data to be visualized, and refers to the node. The algorithm sorts and adapts the weight vectors such that similar data is mapped to the closest node. Each component of the weight vector may be visualized by coloring the grid according to the values of the selected component, and so many dimensions may be visualized at once by displaying several component maps side by side.

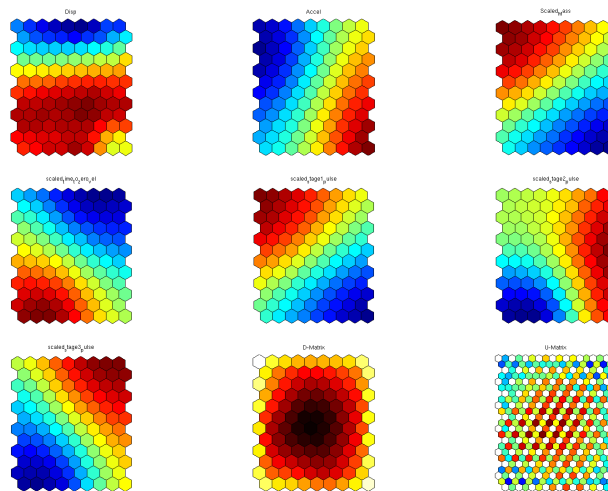


Figure 6: SOM component maps of objectives, constraints, and uniformity measures

A component map of Pareto optimal front data is plotted in Figure 6 and Figure 7. Acceleration, time-to-zero-velocity and mass are in perfect conflict. Displacement objective conflicts with other objectives but has relatively agreeable nature with the acceleration and time-to-zero-velocity. While the constraint on the first stage-pulse agrees closely with the mass, the second and third stage-pulses show conflicting behavior with all objectives. A comparative look at the design variables at the Pareto front reveal some very interesting correlations among design variables and objectives.

1. Thicknesses T2, T5, T6 and T73 are largely in agreement with the acceleration and in-inverse

relation with mass and stage pulse 1.

2. Variation in the thickness T4 is in concert with the time-to-zero-velocity objective.
3. T64 correlates with stage pulse 2, T4 inversely correlates with stage pulse 3.

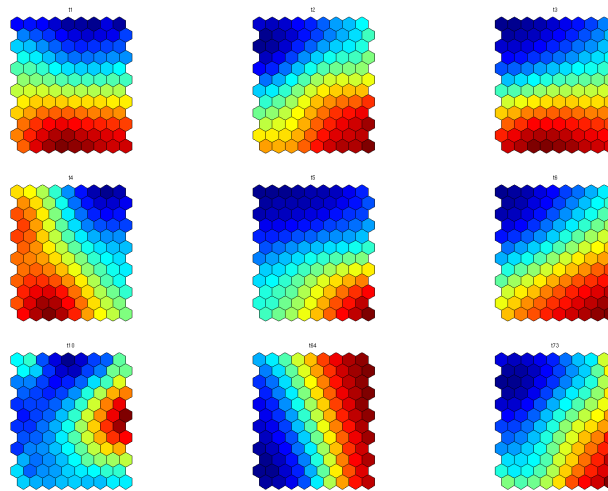


Figure 7: SOM component maps of variables at the Pareto optimal front

## 2.5 Combining all visualization techniques

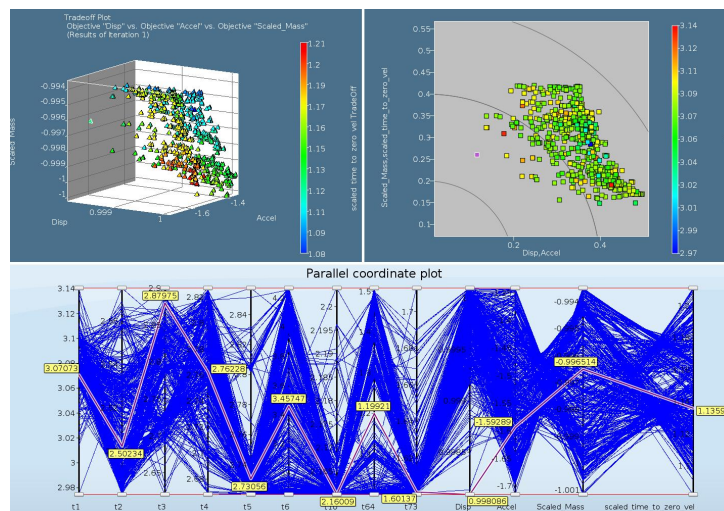


Figure 8: Highlighting a selected design in all graphs

One can take advantage of essential features of different visualization techniques by plotting all of them simultaneously, as shown in Figure 8. The best design from the HRV plot is shown in PCP and Trade-off plot. The cross-plot enables us better understand the trade-off among all designs and the selected design, for example the selected design is a compromise in the acceleration, mass and time-to-zero-velocity while being the best design according to the displacement criterion. Similarly, one can also see that three design variables are near their lower bounds and one design variable is near the upper bound. Also it is obvious from the Trade-off chart that this design is near the discontinuity.

### 3 Conclusions

To rate and compare the presented visualization methods, there are some criteria to look at.

The first aspect is that they differ in the number of dimensions the method is capable to visualize. Another feature could be the ability to detect trends in the Pareto front, i.e. how different variables, objectives, and constraints interact in various regions. The engineer could also be interested in the continuity of the Pareto optimal front, hence the method should identify gaps in the Pareto front. In the end, the engineer needs to select a single design or a few possible designs. Does the method provide this? And last but not least, some visualization methods are more intuitive to the designer than others, and hence more comfortable to use.

These criteria are evaluated for the presented visualization methods and summarized in Table 4.

Table 4: Comparison of visualization methods

Feature	TradeOff	PCP	HRV	SOM
Dimensionality	best with 2/3, can display 5	any number	any number	any number
Trend analysis	weak	weak	no detection	strong detection
Discontinuities	can detect	no detection	no detection	strong detection
Design selection	weak	strong	strong	moderate
Few tradeoffs	moderate	weak	weak	strong
Ease of use	moderate	strong	strong	weak

To get an overall view on an optimization problem, the designer is recommended to combine all the methods and take advantage of each method's capabilities.

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