Verification of cylindrical interference fits under impact loads with LS-Dyna®

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Summary:

Interference fits are a commonly used means to couple shafts and wheels for example. The usual dimensioning is performed by a static verification. As long as the system geometry is not too complicated and the deformation is assumed to be linear elastic, the interference pressure can easily be calculated with the more familiar solutions of the equations of elasticity. The maximum static contact forces can be calculated together with an assumed coefficient of static friction. In order to investigate whether a cylindrical interference fit provides sufficient stability against slip the real loads have to be known. However in various applications this is not the case and the interference fit is subjected to dynamic loads, especially to impact loads. We simulate a model interference fit that is first axially mounted and later also axially loaded. This is a typical case in hydraulic systems. Similar problems occur in gears, e.g. worm gears, especially if there are reverse torques as in many applications.

The crucial number a design engineer seeks is the safety against slip, *S*. A dimensional analysis shows that *S* is dependent on the length *l* of the interference fit, its interference *Z*, the velocity *v* and the mass of the impacting body *m* and the static friction coefficient *μ*. Altogether we find: $S \sim Z\, l\, \mu\, v^{-1}\, m^{-0.5}$. Numerical experiments have shown that the easiest way is to vary the velocity of the impacting body to find the design with minimum safety *S* = 1. The desired safety can then be achieved by simply changing the parameters.

We investigate the influence of different contact types, and find the *OSTS* contact as optimal for the shaft-hub contact. The same way we consider the *NTS* contact as optimal for the shaft-impacting body contact. The results also show that the forces due to an impact are huge and that it is not possible to make an appropriate design without a numerical or experimental analysis.

Keywords:

Interference fit, dynamic load, impact load, contact force, safety against slip

1 Introduction

Interference fits are widely used in mechanical engineering design in order to transmit torsional loads in the first place – particularly in gears –and frequently axial loads, e.g. in hydraulic systems. The general advantage of such joints is the avoidance of notches that cause a reduction in strength, especially when dynamic loads are applied. Also different kinds of material can easily be introduced, as for instance in worm gearings. The general mechanics is easy, the outer diameter of the inner part (shaft, index *S*) is larger than the inner diameter of the outer part (hub, index *H*) – the negative diameter difference is called interference *Z*. *Figure 1* shows the situation. This interference geometry causes a surface pressure *p* that can bear huge torque or axial loads due to static friction. This pressure also causes stresses in the shaft and the body so that *p* and hence *Z* is limited, depending on material and geometry. Thus the dimensioning of an interference fit is an optimization. If *Z* is too small, the result is slip and if it is too large, the result may be excessive stresses.

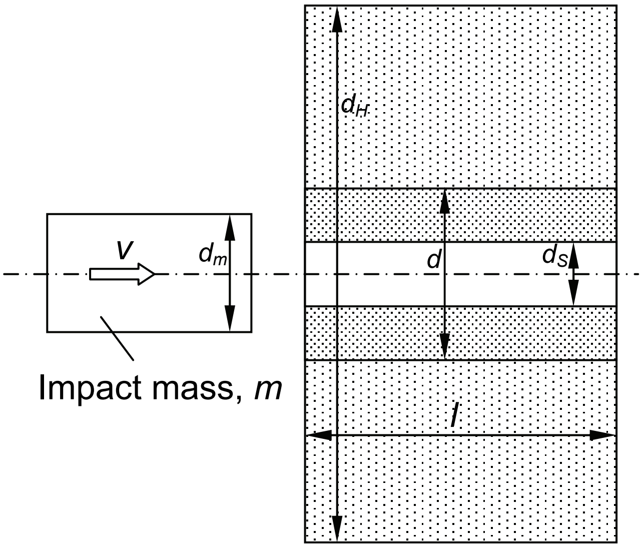

Figure 1: Interference fit with an impact mass

Since for elastic behaviour *p* is linear in *Z*, the dimensioning can be conducted according to DIN 7190 [1]. The underlying formula (1) is an exact solution of the equations for the two-dimensional interference fit [2]. Of course small three-dimensional effects like notches or lubrication holes are usually disregarded. A safety against slip *S* is easily computed since in the static or quasi-static case the loads are given. Moreover in [3] an optimal aspect ratio is introduced. In [2] the interference fit under plastic deformation is described.

$$Z = p \cdot d \cdot \left[\frac{1}{E_H} \left\{ \frac{1+Q_H^2}{1-Q_H^2} + \nu_B \right\} + \frac{1}{E_S} \left\{ \frac{1+Q_S^2}{1-Q_S^2} - \nu_I \right\} \right] \tag{1}$$

In (1) the parameters *Q* represent the diameter ratios:

$$Q_H = \frac{d}{d_H} \qquad Q_S = \frac{d_S}{d} \tag{2}$$

The maximum von-Mises-stress is at the inner diameter of the shaft. According to [1] and [2] it is given by:

$$\sigma_v = \left| -p \right| \cdot \frac{2}{1-Q_S^2} \tag{3}$$

Frequently, e.g. in valves or other applications, the loads are dynamic or even of impact type. As with other impact problems the load depends on the masses, velocities and the material properties and can not be easily calculated. Therefore we use LS-DYNA® to simulate a simple impact on an interference fit to predict the safety *S* of the fit. There is little literature on the dynamic behaviour of interference fits

– mostly the dynamic bending loads are considered [4] and [5]. Würtz [6] describes the mounting of such fits with impact forces as generally unclear and dependent on the energy that is applied to the fit.
There are two different ways of mounting an interference fit: radial and axial. In radial mounting the interference must temporarily be zero, for instance by subjecting the outer part to a higher temperature. In axial mounting two chamfers are necessary and the shaft is inserted with high axial loads. In axial mounting the surface roughness is smoothed by this process and thus the remaining interference is reduced by *R,* (4) gives a good estimate, R_z are the surface roughnesses of the two surfaces. However the structure of the surfaces cannot be considered in an analysis with LS-DYNA®. In our model *Z* is understood as an effective interference “after” mounting.

$$R \approx 0.8 \cdot \left(R_{zS} + R_{zH}\right) \tag{4}$$

2 Theory and Dimensional Analysis

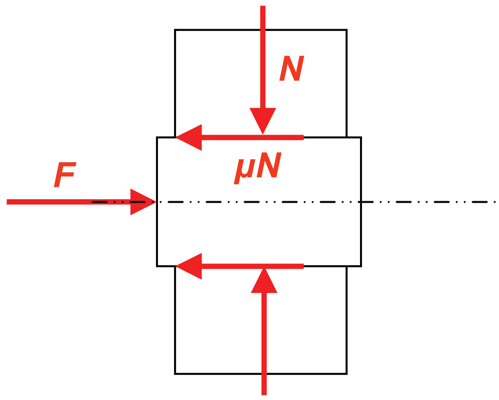

Figure 2: Forces on the interference fit

Figure 2 indicates the parameters that determine the safety *S* against slip. That means – assuming Coulomb’s friction:

$$S = \frac{\mu N}{F} \tag{5}$$

The resulting normal force is proportional to the pressure in the fit:

$$N = \pi \cdot p \cdot l \cdot d \tag{6}$$

With (1) the pressure can be represented by:

$$p = \frac{Z}{K \cdot d} \tag{7}$$

K is a design parameter that includes material properties as well as diameter ratios. Thus we find:

$$S = \frac{\mu \cdot \pi \cdot l \cdot Z}{F \cdot K} \tag{8}$$

Therefore the main obstacle to determine the safety against slip is the uncertainty in terms of *F*, which depends on mass and velocity, but also on material properties and geometry. To get a bit closer we consider the momentum during the impact. As a reasonable guess we assume the force *F*(*t*) to be sinusoidal:

$$F(t) = F_0 \cdot \sin(\omega t) \tag{9}$$

The frequency ω depends on the size of the impacting mass *m* and the stiffness *c* of the system and it is related to t_S.

$$\omega^2 = \frac{c}{m} \qquad t_S = \frac{\pi}{\omega} \tag{10}$$

So it is a half-sine that is under consideration during t_S. Therefore we obtain:

$$t_S = \pi\sqrt{\frac{m}{c}} \tag{11}$$

We investigate the impact equations for the case that the combined mass of shaft and hub is fixed, its velocity before and after the impact is zero as long as the interference fit is safe. Therefore for the impact number *e* we obtain

$$e = -\frac{v'}{v} \tag{12}$$

In (12) *v'* is the velocity of the impacting mass after the impact. Thus altogether for the force *F*:

$$F' = \int_0^{t_S} F(t) = \left|m(\bar{v} - v)\right| = \left|m(-ev - v)\right| = m(1+e)v \tag{13}$$

Obviously the worst-case-scenario is *e = 1*, meaning that the involved parts are infinitely hard. That results in:

$$F' = 2m \cdot v \tag{14}$$

Because of the half-sine the resulting force is:

$$F = \frac{\pi}{2 \cdot t_s} F' = \frac{\omega}{2} F' = \sqrt{\frac{c}{m}} \cdot \frac{F'}{2} = \sqrt{c \cdot m} \cdot v \tag{15}$$

Although these formulas reflect certain assumptions, we can, however, consider the safety against slip to be proportional to the involved parameters as follows:

$$S \sim \frac{\mu \cdot l \cdot Z}{\sqrt{c \cdot m} \cdot v \cdot K} \tag{16}$$

We note that *K ~ E* and *c ~ E*, but since both parameters also include geometry, (16) means we can either vary *μ*, *l*, *Z* or *v* linearly or the mass *m* in order to examine the reliability of the fit. This is important because the variation of either parameter could cause different numerical obstacles, especially varying *Z* and *μ* can cause contact problems. The idea is that if an axial motion will occur in the simulation at a certain interference *Z**, than one has to choose a *Z* that equals *S*-times *Z**. Of course this can be performed using *μ*, *v*, *m* or *l* respectively as variation parameters. Thus the determination of *S* is an iterative parameter study. The single parameter to be changed is φ:

$$\varphi = \frac{\mu \cdot l \cdot Z}{\sqrt{m} \cdot v} \tag{17}$$

3 Model

The model we used for simulation is shown in *figure 3*. The mass *m* of the impacting cylindrical part is *0.075 kg* and its “design velocity” is *1 m/s*. We assume axial mounting of the interference fit and also simulate this. After mounting there is a pause to make sure there is no kinetic energy left in the system before the impact. The whole process takes *25 milliseconds [ms]* – the mounting takes *10 ms*, the following pause is *5 ms*, before the motion of the impact mass starts.

According to figure 3 the parameters in (1) are Q_H = 0.32 and Q_S = 0.375 respectively. Comparing the von-Mises-stress of (3) with an assumed yield stress of R_e = 900 MPa the maximum pressure is 387 MPa and the maximum interference is 79µm. The analysis is performed with an interference of 54 µm however and a static friction coefficient of *μ* = 0.15. The irrelevant dynamic friction coefficient is set to 0.005, and we choose a decay coefficient of 0.00008 – therefore the mounting process can easily be performed. The interference fit is not validated if an axial motion of the shaft occurs. The Young’s moduli are E_H = 200 GPa (Steel) and E_S = 216 GPa (Vanadis).

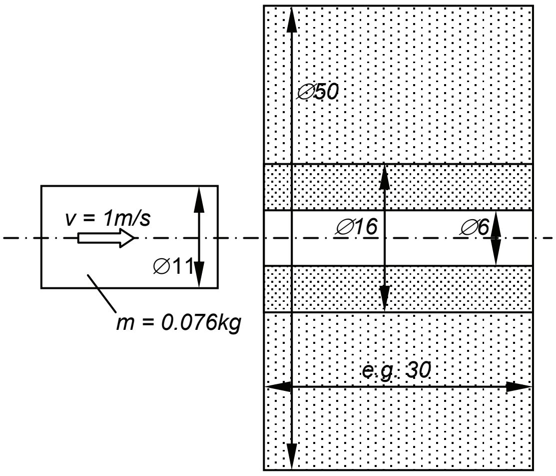

Figure 3: *Model Geometry*

The contact between shaft and hub is a one-way-surface-to-surface contact, *OSTS*. The contact between the impacting mass and the shaft is a node-to-surface contact, *NTS*. In the contact card we use the following options: ABC, SOFT = 1, SOFSCL = 0.1. In the Hourglassing card we use *IHQ* = 6 and *QM* = 1.0. Thus hourglassing is not a problem. However, we performed a couple of simulations with fully integrated elements. *Figure 4* shows the elements of the system.

4 Simulation and Results

Of course it is reasonable not to alter the designed geometry – that is the length *l*. Therefore we have left static friction coefficient *μ*, the impact speed *v* or mass *m* or the interference *Z* to examine the safety against slip. Actually we experienced contact problems in both using a very high static friction coefficient and a large interferences that cause numerical deformations of the shaft. To avoid this we vary the impact speed.

The simulation with LS-DYNA® is performed on the Sun Fire X 2200 High-Performance-Computing Cluster at Mannheim University of Applied Sciences. The cluster has 14 nodes, each 2 dual core (AMD Opteron) with 16 GB RAM per node and a speed of 2.6 GHz. The nodes are connected via Infiniband at 20 Gbit/s.

The typical behaviour of such an interference fit under impact load is illustrated by the figures 5, 6 and 7. Figure 5 shows a validated fit, the velocity of the impacting mass is 0.125 m/s. There is no slip at all. Figure 6 shows the moment of the impact of a failed fit, the velocity of the impacting mass is 1.0 m/s. There is also a notable elastic deformation of the shaft. Figure 7 shows the same fit, but 0.6 ms after the impact. The full slip then already happened and is 0.08 mm. Because of friction the shaft is at rest then. Figures 8 and 9 show a large slip brought on by an impact velocity of 2 m/s. The slip in this case is 0.56 mm. Thus different velocities give different slips. The numerical experiments show that in the interference fit under consideration slip will occur if the velocity of the impacting mass is larger than 0.58 m/s. Therefore we can also give a guess of the impact time t_S. However assuming a half-sine for $F(t)$ over t_S. From (1), (14) and (15) we obtain:

$$t_S = \frac{\pi \cdot F'}{2 \cdot F_0} = \frac{m \cdot v}{\mu \cdot p \cdot l \cdot d} \qquad (18)$$

With our data we obtain t_S = 2.4 μs. According to [7] the smallest time step of our model will be automatically set to 0.086 μs, because the element size is 0.5 mm, so the element size can be considered as sufficiently small.

The CPU-time is about 2000 s when using fully-integrated elements.

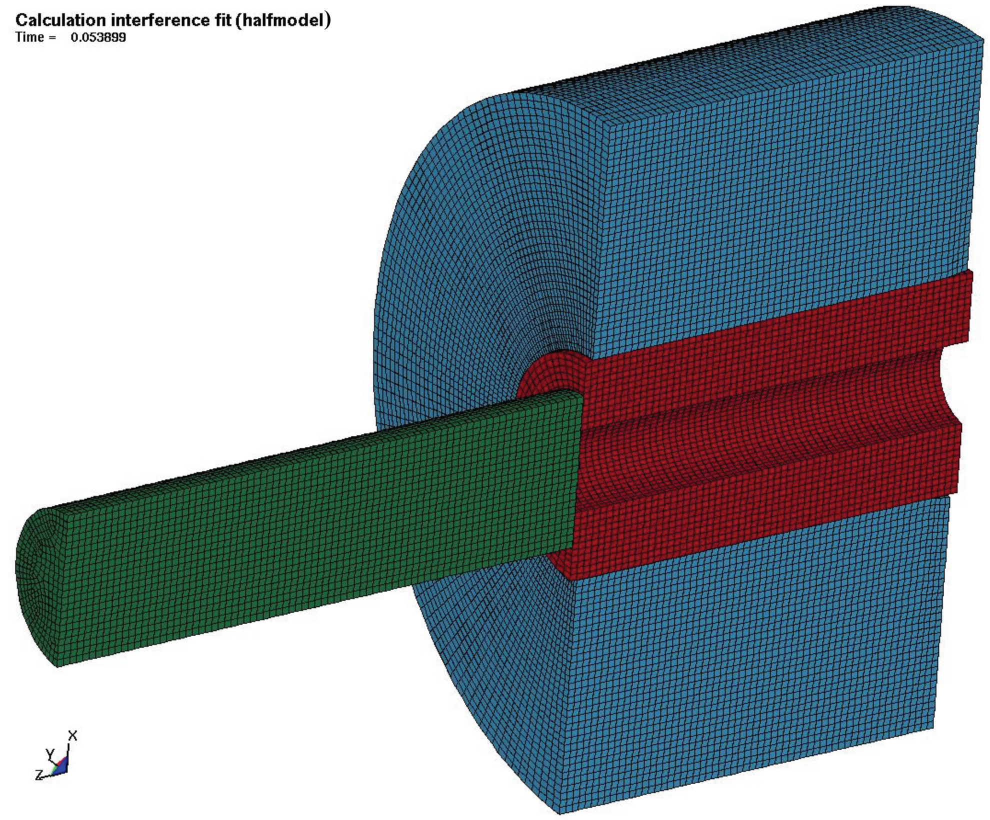

Figure 4: Elements of the simulated system

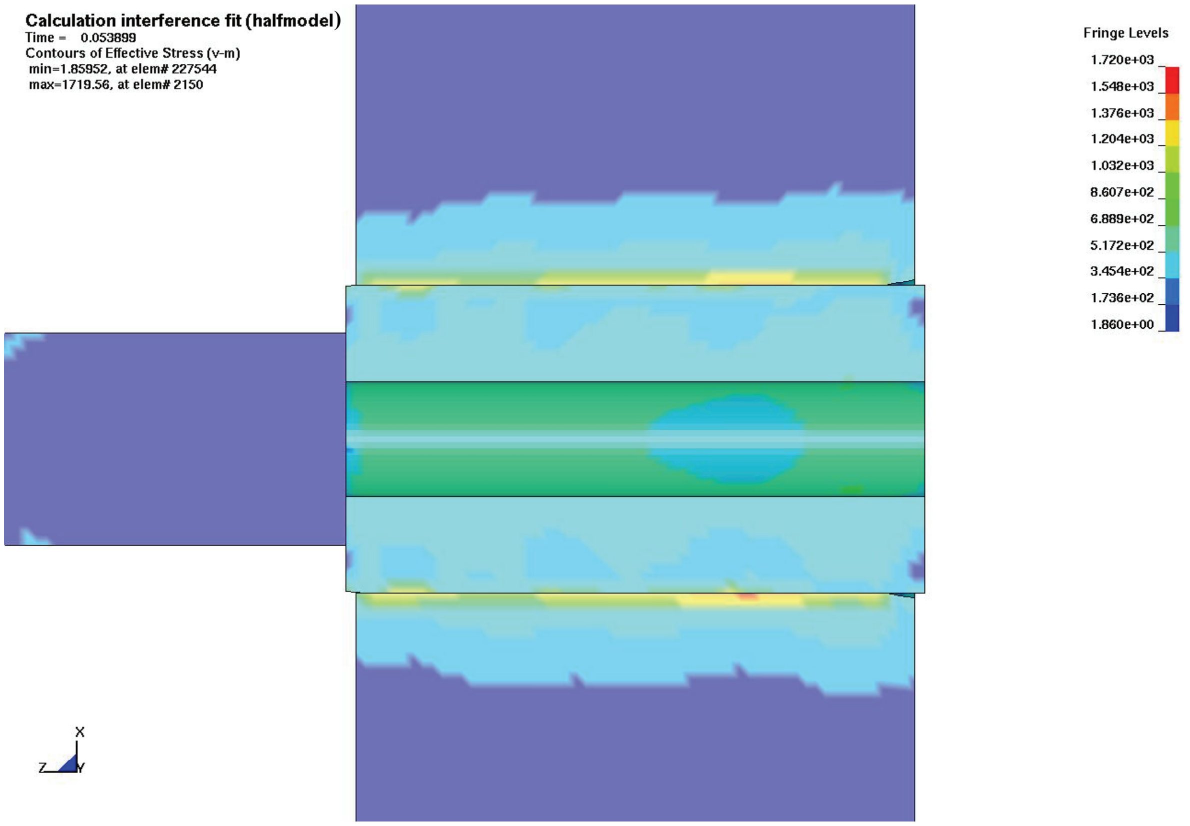

Figure 5: Interference fit without slip (validated fit), simulated with v = 0.125 m/s

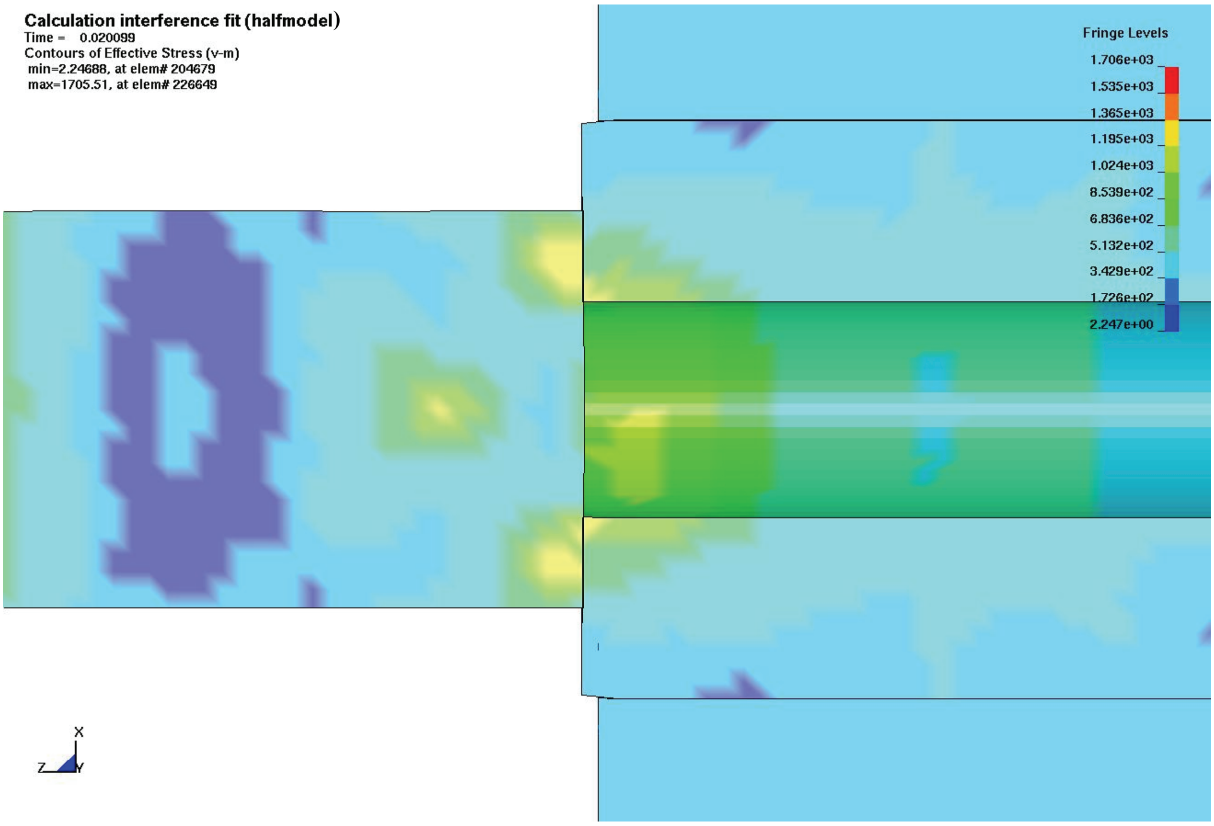

Figure 6: Interference fit with slip (failed fit), simulated with v = 1.0 m/s. The moment of impact.

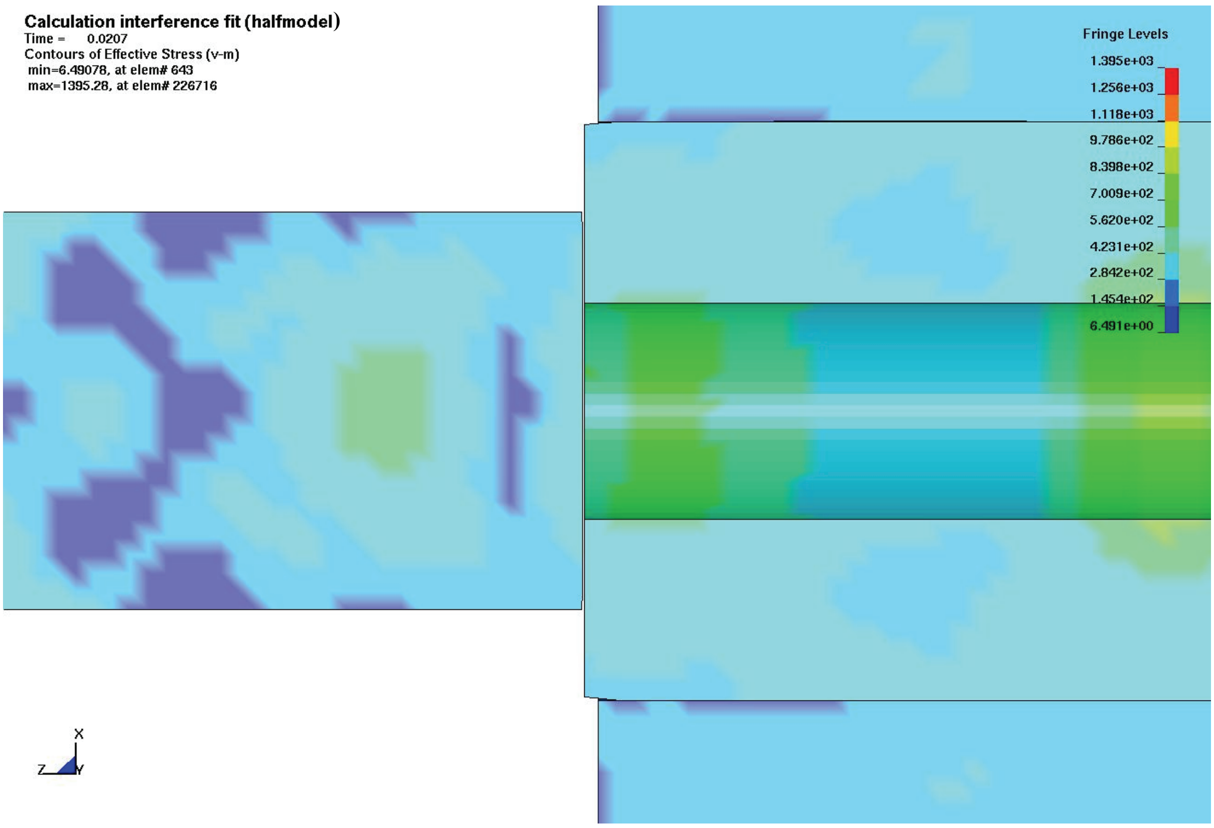

Figure 7: Interference fit with slip (failed fit), simulated with v = 1.0 m/s. 0.6 ms after the impact. The slip is 0.08 mm.

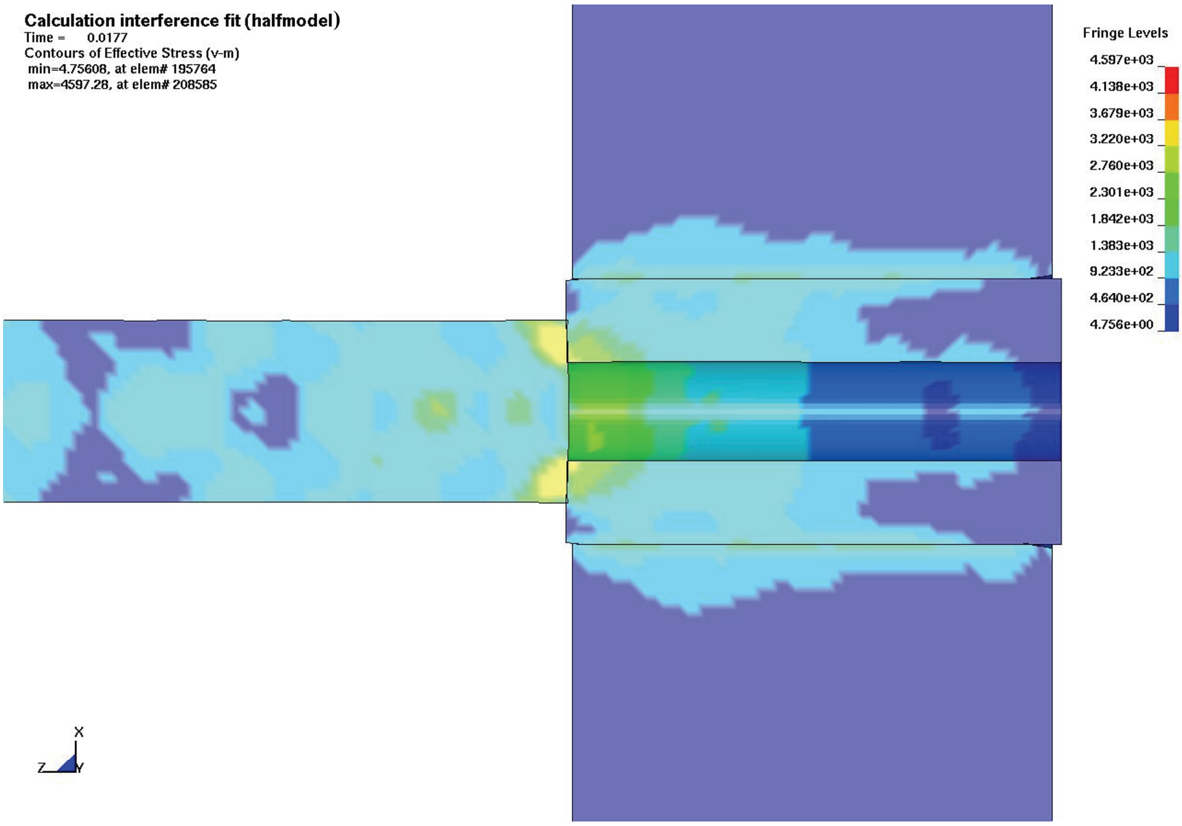

Figure 8: *Interference fit with slip (failed fit), simulated with v = 2.0 m/s. The moment of impact.*

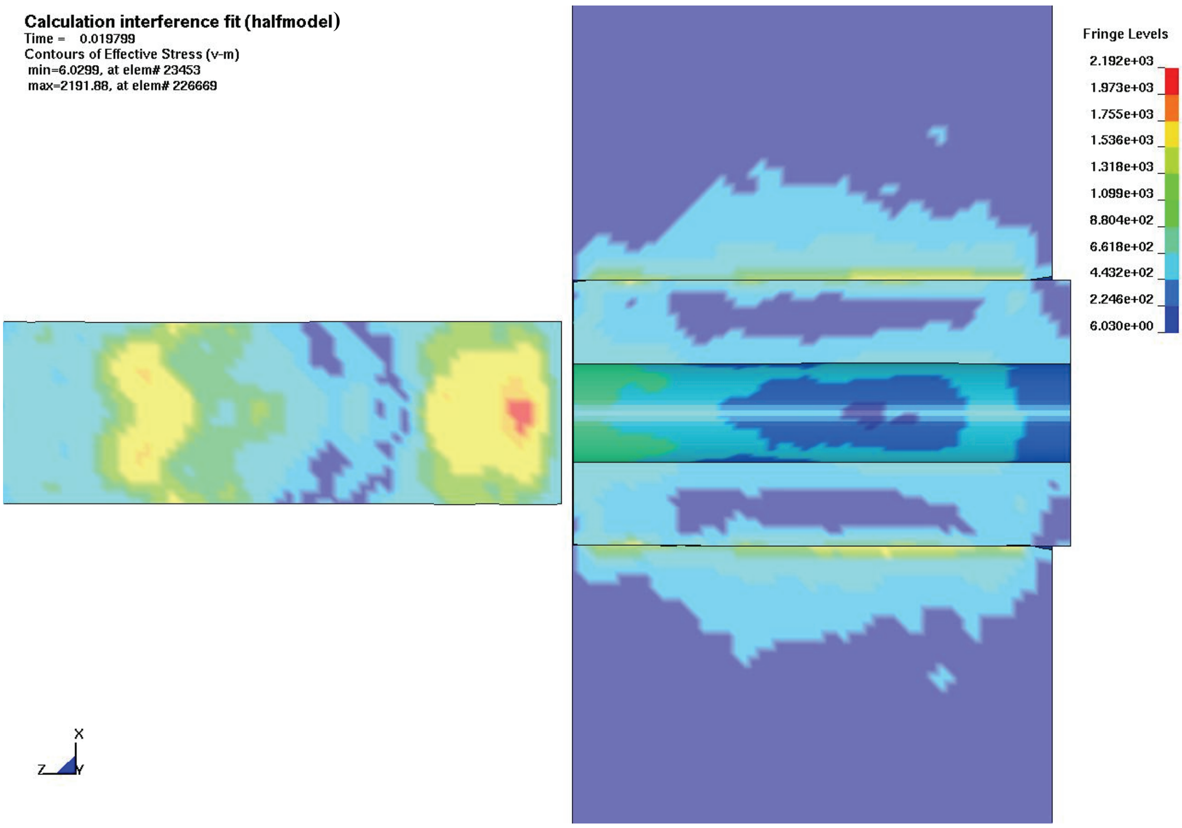

Figure 9: *Interference fit with slip (failed fit), simulated with v = 2.0 m/s. 2 ms after the impact. The slip is 0.56 mm.*

5 Conclusions

The interference fit under impact loads can be simulated with LS-DYNA. In an iterative way it is possible to determine safety against slip. Because of possible obstacles in changing contact parameters ivarying the velocity is recommended. The obtained results should be validated by a set of experiments. Because of the vast number of parameters it seems to be hazardous to suggest a design formula that can be used to compute *S* as easy as in the case of a pure static load.

6 Literature

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Appendix: Notation

Z	Interference
p	Pressure
d	Fit diameter
d_H	Outer diameter of the hub
d_S	Inner diameter of the shaft
$E_{H,S}$	Young's modulus of hub and shaft
$Q_{H,S}$	Diameter ratio
σ_V	Von-Mises-stress
R	Reduction of interference
R_z	Roughness
S	Safety against slip
N	Normal force
μ	Static friction coefficient
l	Length
K	Parameter
F, F_0	Load
F'	Time integral of F
t	Time
t_S	Duration of the impact
m	Mass
c	Stiffness
ω	Natural frequency
v, v'	Velocity
e	Impact number
R_e	Yield stress
φ	Similarity parameter